Dimensional analysis

The Buckingham π Theorem - If there are m physical quantities containing r fundamental units, then there are $\pi = m - r$ dimensionless groups that are independent.

Procedure:
1. Write down all relevant physical quantities and their fundamental units.
2. Determine π.
3. Write down the π dimensionless groups.

Example: Flow through a circular pipe

$$\frac{dP}{dx} = f(Q, R, \mu) \qquad m = 4$$

$$\frac{dP}{dx} \quad \left(\frac{\text{dynes}}{\text{cm}^3} = \frac{\text{g}}{\text{cm}^2\text{s}^2}\right)$$

$$Q \quad (\text{cm}^3/\text{s})$$

$$R \quad (\text{cm})$$

$$\mu \quad \left(\frac{\text{dynes}}{\text{cm}^2}\text{s} = \frac{\text{g}}{\text{cm s}}\right) \qquad \text{g, cm, s} \quad r = 3$$

$$\therefore \pi = m - r = 1$$

$$\frac{1}{\mu}\frac{dP}{dx} \quad \left(\frac{1}{\text{cm s}}\right) \qquad \text{only way to get rid of grams}$$

$$\frac{1}{\mu Q}\frac{dP}{dx} \quad \left(\frac{1}{\text{cm}^4}\right) \qquad \text{only way to get rid of seconds}$$

$$\frac{R^4}{\mu Q}\frac{dP}{dx} \quad \text{(dimensionless)}$$

$$\text{Dimensional Analysis} \quad \Rightarrow \quad \frac{dP}{dx} \sim \frac{\mu Q}{R^4}$$

Dimensional analysis

$$\text{Dimensional Analysis} \quad \Rightarrow \quad \frac{dP}{dx} \sim \frac{\mu Q}{R^4}$$

Not as useful as the exact result:

$$\frac{\Delta P}{L} = \frac{8}{\pi}\frac{\mu Q}{R^4}$$

because exact result has the prefactor.

How did we get so much information? **We cheated!**

Used "experience" to guess that ΔP and L would only enter as $dP/dx = \Delta P/L$, and that the velocity v would only enter though Q.

Dimensional analysis
EXAMPLE: FLOW IN AN EXTRUDER

$$\frac{dP}{dx} = f(Q, D_s, h_s, N_s, \phi_s, \mu) \qquad \left(\frac{\mathrm{g}}{\mathrm{cm}^2\mathrm{s}^2}\right)$$

Q (cm^3/s) volumetric flow rate

D_s (cm) barrel diameter

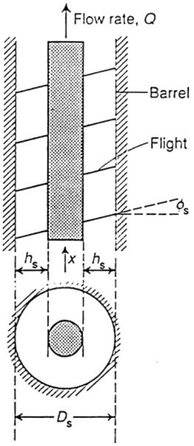
h_s (cm) flight depth

N_s (1/s) screw rotation rate

ϕ_s (dimensionless) helical angle of screw

μ (g/cm s) viscosity

$$m = 7, \quad r = 3 \quad \Rightarrow \pi = m - r = 4$$



Four dimensionless groups

$$\phi_s, \frac{D_s}{h_s}, \frac{Q}{N_s D_s^3}$$

$$\frac{1}{\mu}\frac{dP}{dx}\left(\frac{1}{\mathrm{cm\ s}}\right) \qquad \text{(cancels grams)}$$

$$\frac{D_s}{\mu N_s}\frac{dP}{dx} \qquad \text{(dimensionless pressure gradient)}$$

Dimensional analysis
EXAMPLE: FLOW IN AN EXTRUDER

$$\frac{D_s}{\mu N_s}\frac{dP}{dx} = f\left(\phi_s, D_s/h_s, Q/(N_s D_s^3)\right)$$

Optimum $\qquad \phi_s = 23^\circ \qquad$ (universally utilized)

Most extruders are geometrically similar and designed to provide a certain pressure at the die:

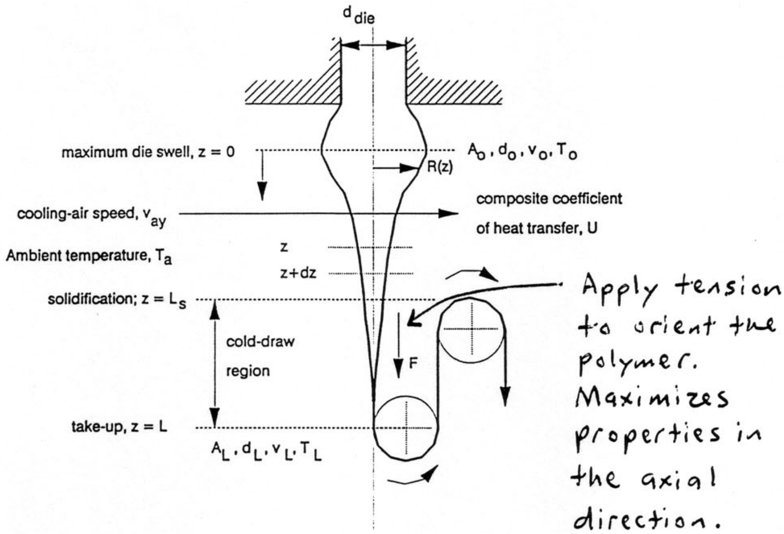
$$\therefore \quad \frac{D_s}{h_s} \cong \text{ constant}$$

$$\therefore \quad \frac{D_s}{\mu N_s}\frac{dP}{dx} \cong \text{ constant}$$

If other three dimensionless groups are constant, then the last one must also be constant.

Expect $\qquad Q \sim N_s D_s^3 \qquad$ (not obvious)

Melt Fiber-Spinning



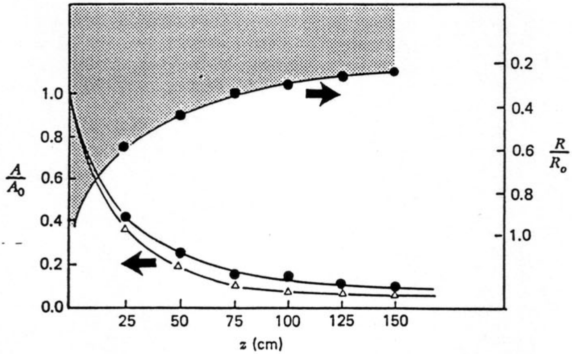


Fig. 15.1 Melt strand area and radius profiles in the melt drawdown region: ●, nylon 6 at 265°C and takeup velocity of 300 m/min; △, PP at 262°C and takeup velocity of 350 m/min. (Reprinted with permission from H. F. Mark, in *Rheology*, Vol. 4, F. R. Eirich, ed., Academic Press, New York, 1969.)

Melt vs. Solution Fiber-Spinning

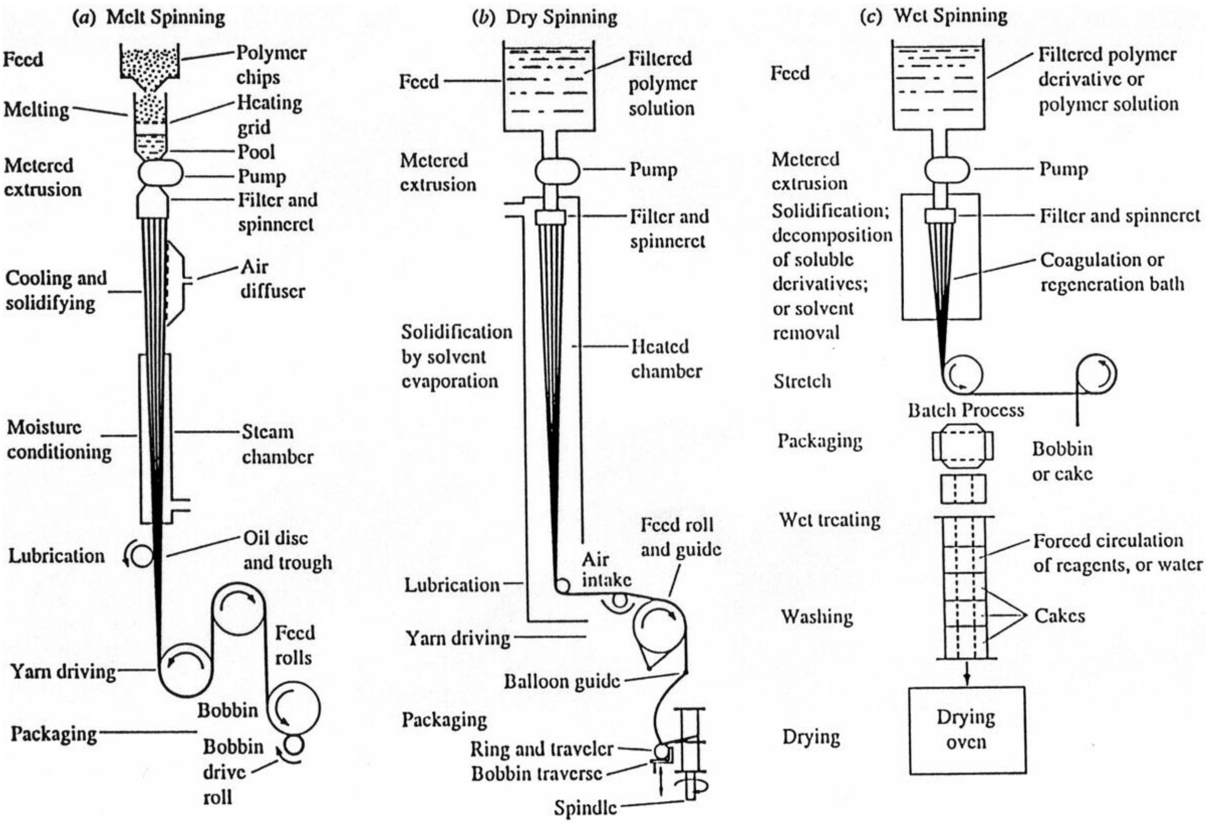
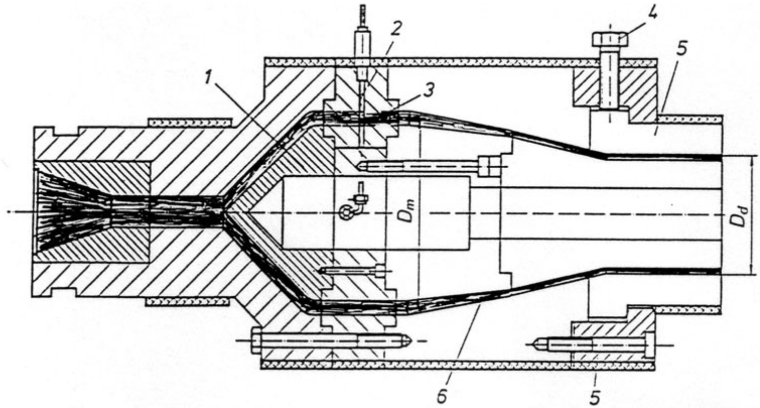


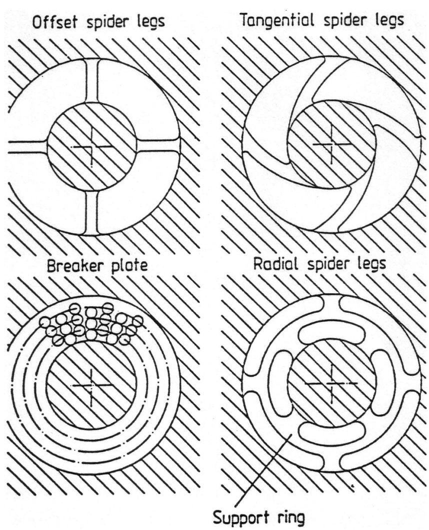
Fig. 11-1 Schematic diagrams of spinning processes [1].

PIPE EXTRUSION: Die Swell is Crucial!



Mandrel support die (principle). *1* Mandrel tip, *2* Mandrel support, *3* Spider leg, *4* Centering screw, *5* Die ring, *6* Relaxation zone

Mandrel Supports:



Profile Extrusion

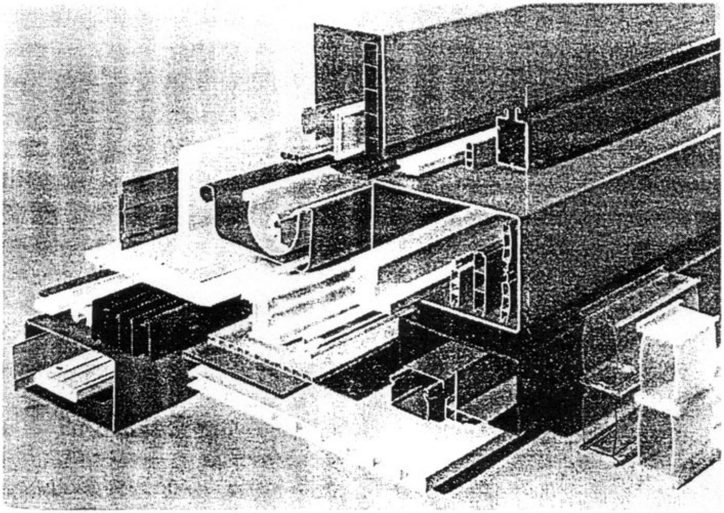
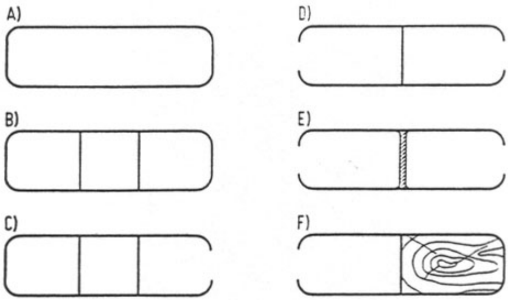
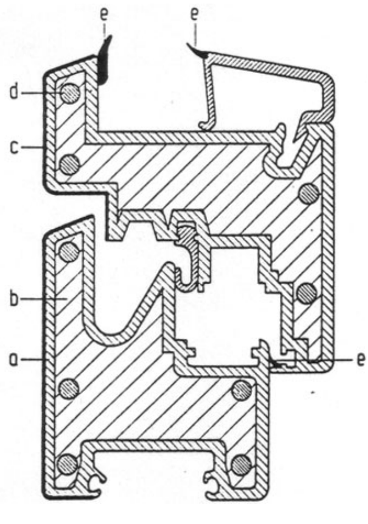


Figure 26. Extruded profiles





Sheet Processing
SHEET EXTRUSION

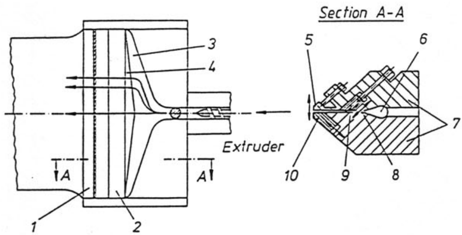


Fig. 5.11 Flat slit die for film extrusion. *1* Lip, *2* Choker bar, *3* Manifold, *4* Island, *5* Flex Lip, *6* Manifold, *7* Body of the die, *8* Land, *9* Choker bar (locally adjustable), *10* Lip (adjustable)

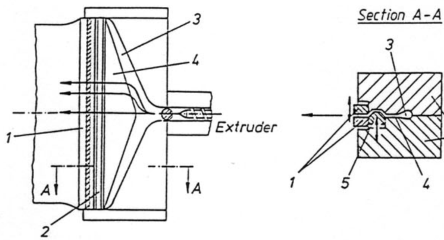
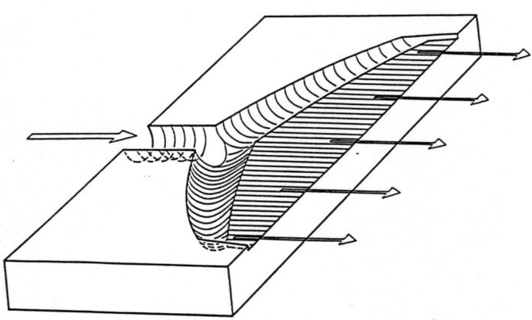


Fig. 5.12 Flat slit die for the extrusion of sheets. *1* Lip, *2* Choker bar, *3* Manifold, *4* Island, *5* Choker bar (locally adjustable), *6* Body of the die



Principle of the coathanger manifold

Sheet Processing
CLAM SHELLING IN A SHEET DIE

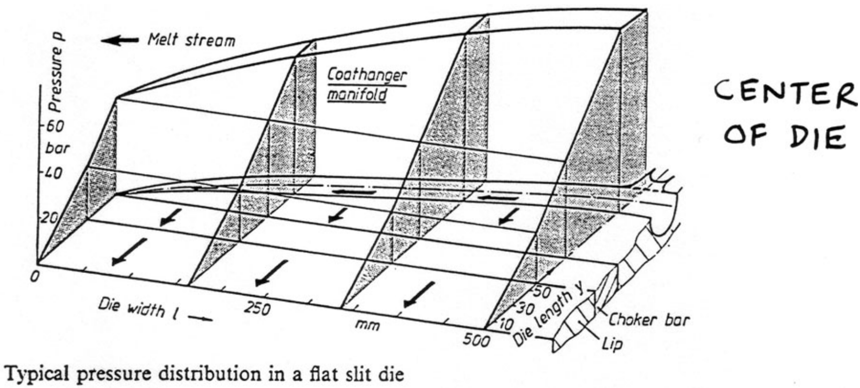


Figure 1: Pressure is very high (100atm) at the center of the die.

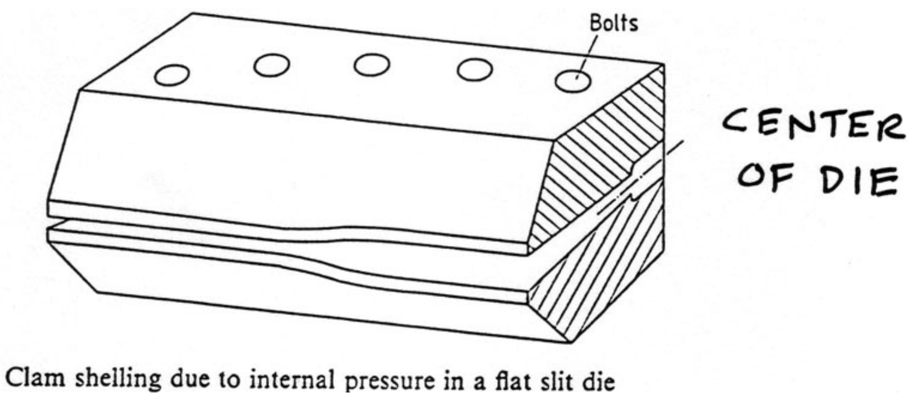
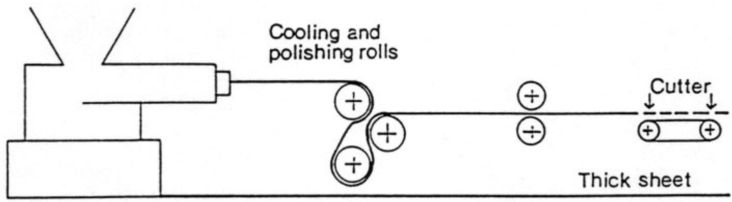
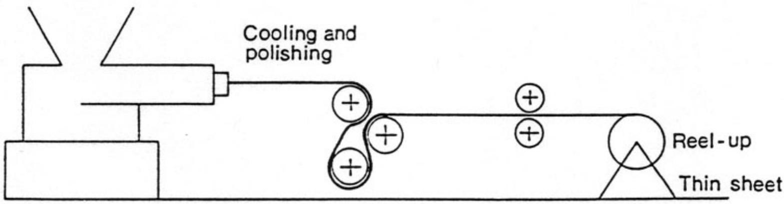


Figure 2: High pressure causes die to creep. After extensive extrusion, sheet will be thicker in the center.

Sheet Processing
POST-DIE SHEET FORMING



(a)



(b)

Calendering Thin Sheet:

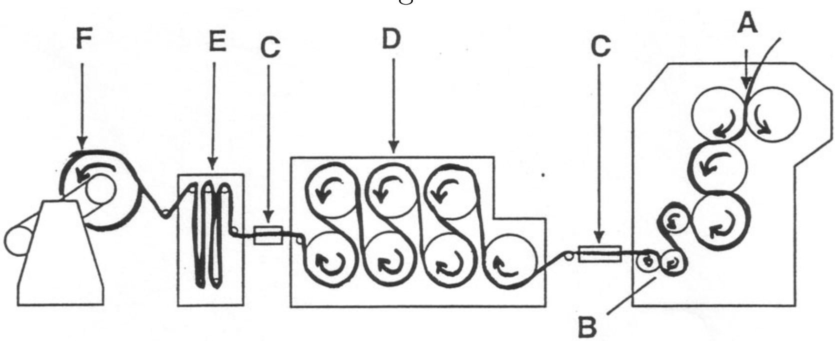


Figure 1.5 Schematic view of an inverted "L" calender plant for the production of plastic sheeting. Molten polymer is dropped into the calender system at **A**. Continuous sheet is formed by passing the melt between the rolls. One of the surfaces is given texture by an embossing roll, **B**. Thickness gauges, **C**, provide data for control of the process. The sheet is cooled, **D**, tensioned using wind-up accumulators, **E**, and accumulated on the wind-up roll, **F** (Holmes-Walker, 1975).

COEXTRUSION LAYER PACKS

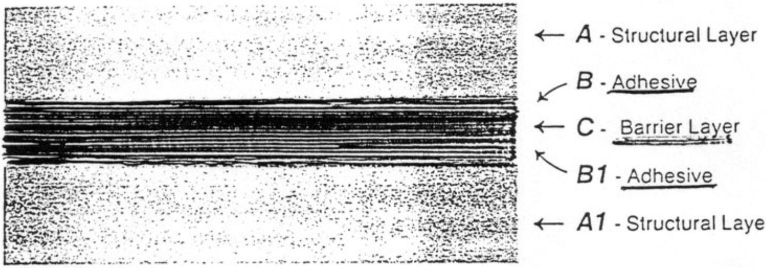


Figure 5-2 Typical structure of a five-layer coextrusion. Courtesy: Dow Chemical Company.

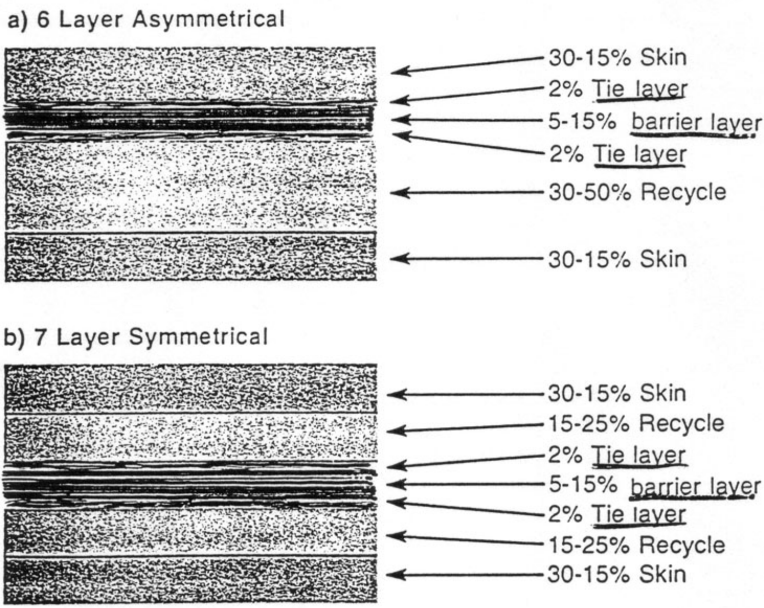


Figure 5-3 Typical coextruded structures containing recycled material in additional layers. (a) Six-layer assymetrical structure. (b) Seven-layer symmetrical structure. Courtesy: Dow Chemical Company.

COEXTRUSION
BARRIER POLYMERS

Table 5-1 Barrier Properties of Commercial Polymers

Polymer	Transmission, cc mil/100 in.²/24 h — Oxygen 25°C, 65% RH	Moisture Vapor 40°C, 90% RH
EVOH	0.05–0.18	1.4–5.4
PVDC	0.15–0.90	0.1–0.2
Acrylonitrile	0.80	5.0
Amorphous nylon	0.74–2.0	—
Oriented PET	2.60	1.2
Oriented nylon	2.10	9.0
Rigid PVC	14.0	3.0
LDPE	420	1.0–1.5
HDPE	150	0.4
PP	150	0.69
PS	350	7–10

Source: EVAL Co. of America. *Plastics Packaging* (July/August 1988): 19–21

EVOH (ethylene-vinyl alcohol) random copolymer with 30%E
PVDC (polyvinylidene chloride) [CH_2CCl_2]

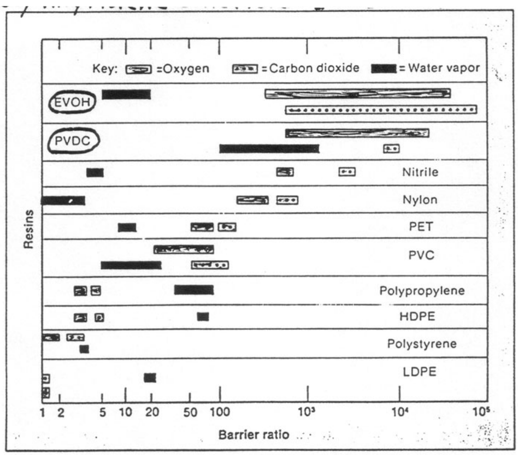


Figure 5-5 Comparison of barrier properties of various polymers used in packaging. Barrier ratio-thickness ratio to reach equivalent barrier. Courtesy: *Plastics Engineering* (May 1986).

COEXTRUSION

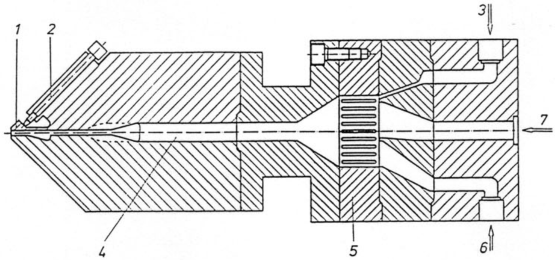
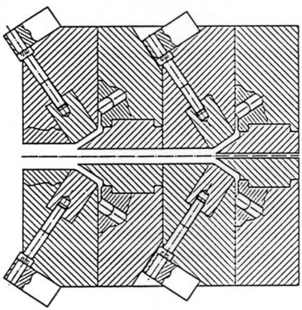


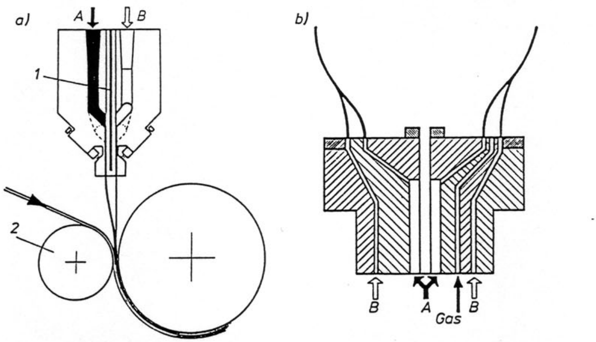
Fig. 6.2 Feed block (System Dow) for bringing melt streams together before the flat slit die. *1* Flex lip, *2* Pressure bolt, *3* Cover layer material, *4* Melt channel with a flow restrictor, *5* Adapter, *6* Base layer material, *7* Main layer material



Used to form a thin barrier layer in packaging applications

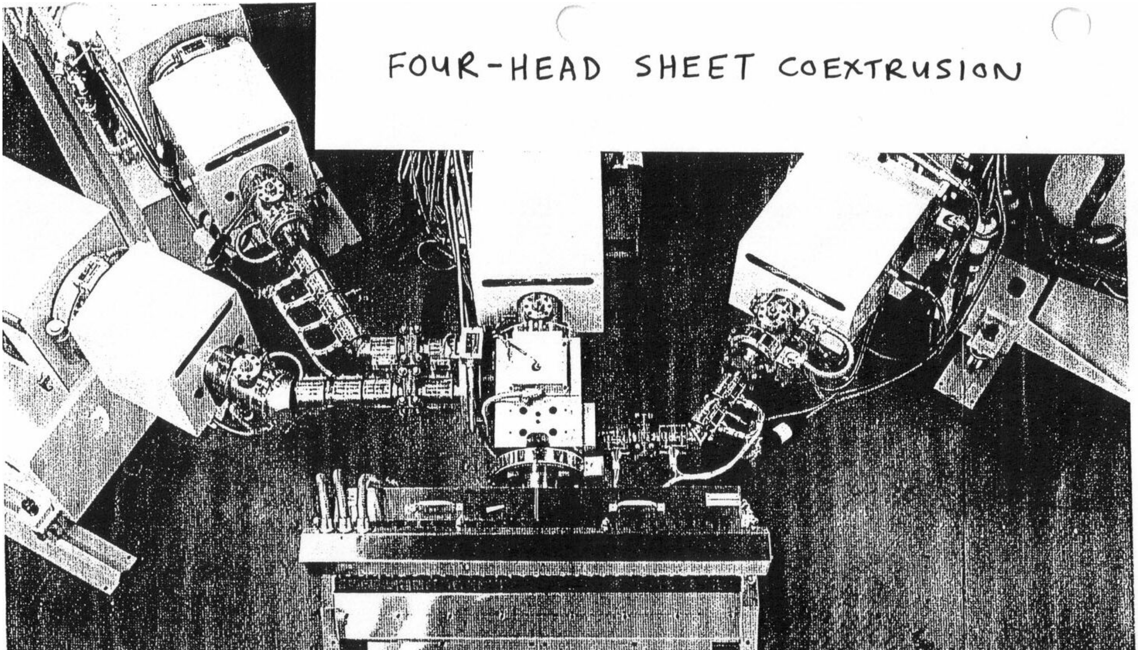
Ex. Nylon / Polypropylene (O_2 diffuses rapidly in PP)

Fig. 6.3 Flat slit die with a sliding adapter (System Reifenhäuser)



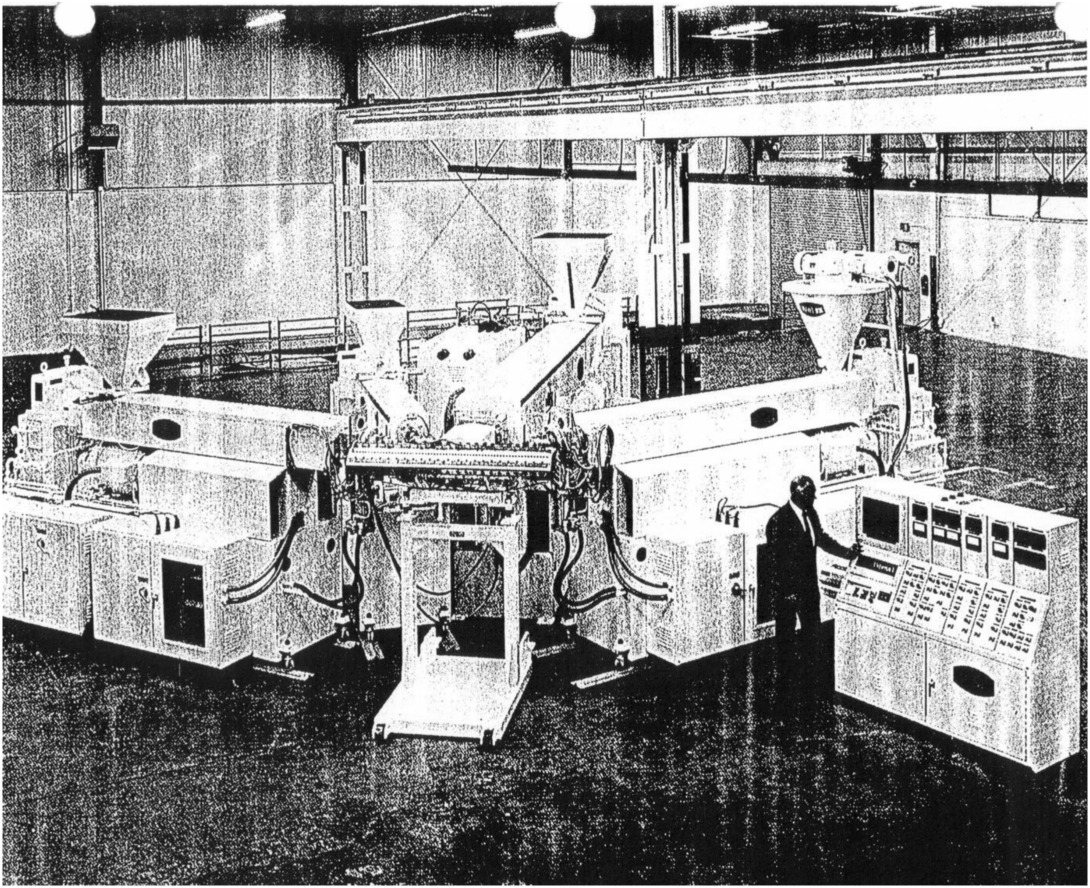
Dual slot coextrusion dies, a) Flat slit die, *1* Heat separation, *2* Pressure roll, b) Blown film die

COEXTRUSION



FOUR-HEAD SHEET COEXTRUSION

COEXTRUSION



FILM BLOWING

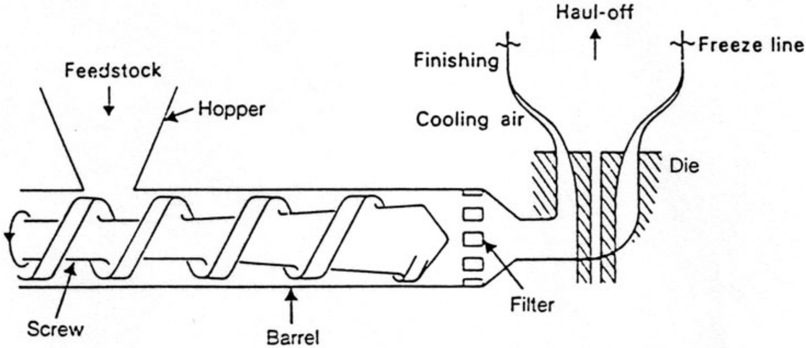


Fig. 7.1. Extruder and die in a film blowing unit (after Cogswell).

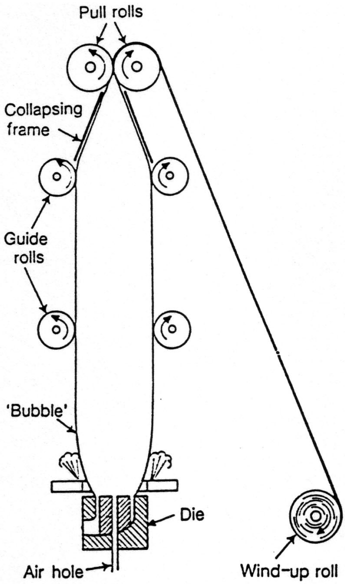


Fig. 7.2. Die, 'bubble', and take-off equipment in a film blowing unit.

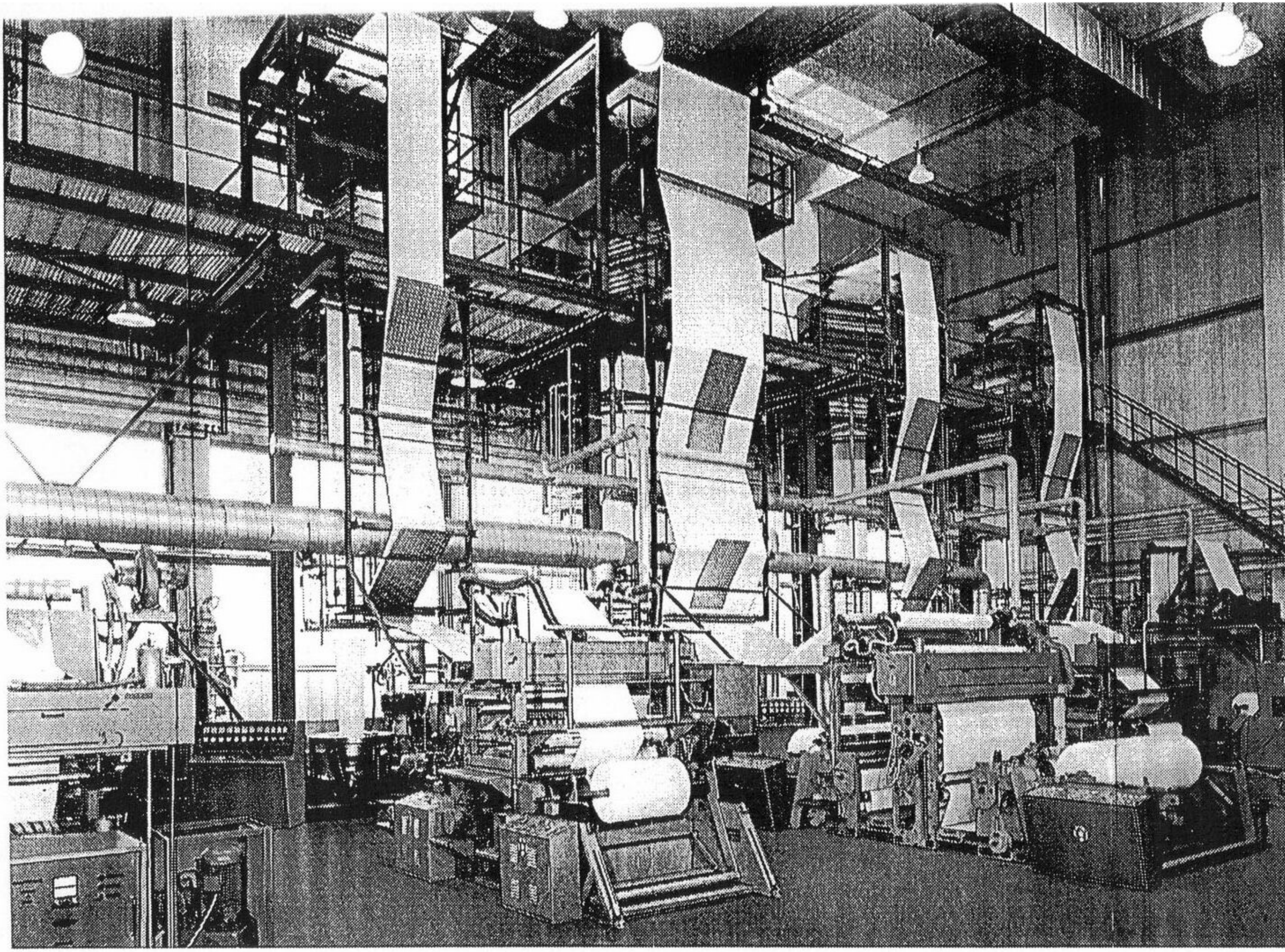
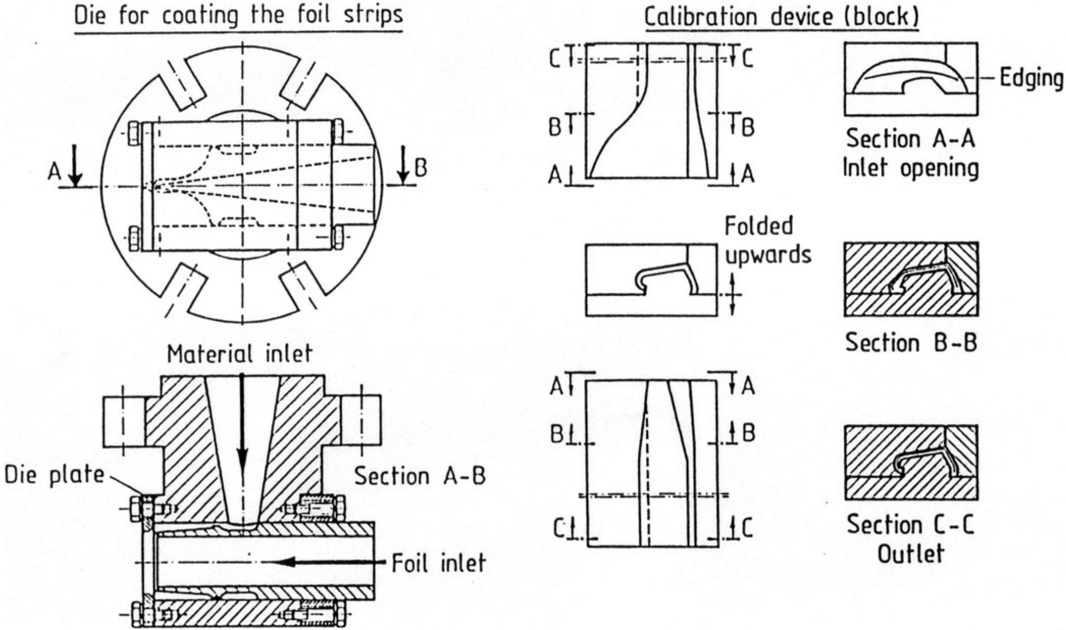
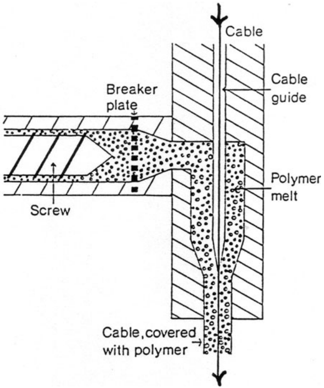


Figure 37. High-output line for LDPE blown film. (Photo: Windmöller und Hölscher, Lengerich, We
many)



Die and calibration device for the extrusion of edging (CAB coated aluminium foil]

Wire Coating



Combination of Poiseuille flow (pressure driven) and Couette (drag) flow due to the moving wire.

More on Wire-Coating in the NEXT LECTURE

Sheet Processing
THERMOFORMING

R.G. Griskey, Polymer Process Engineering, chapter 10

J. Florian, Practical Thermoforming (Marcel Dekker, 1987)

Extrude a Sheet, Clamp it, Heat it up, and ...

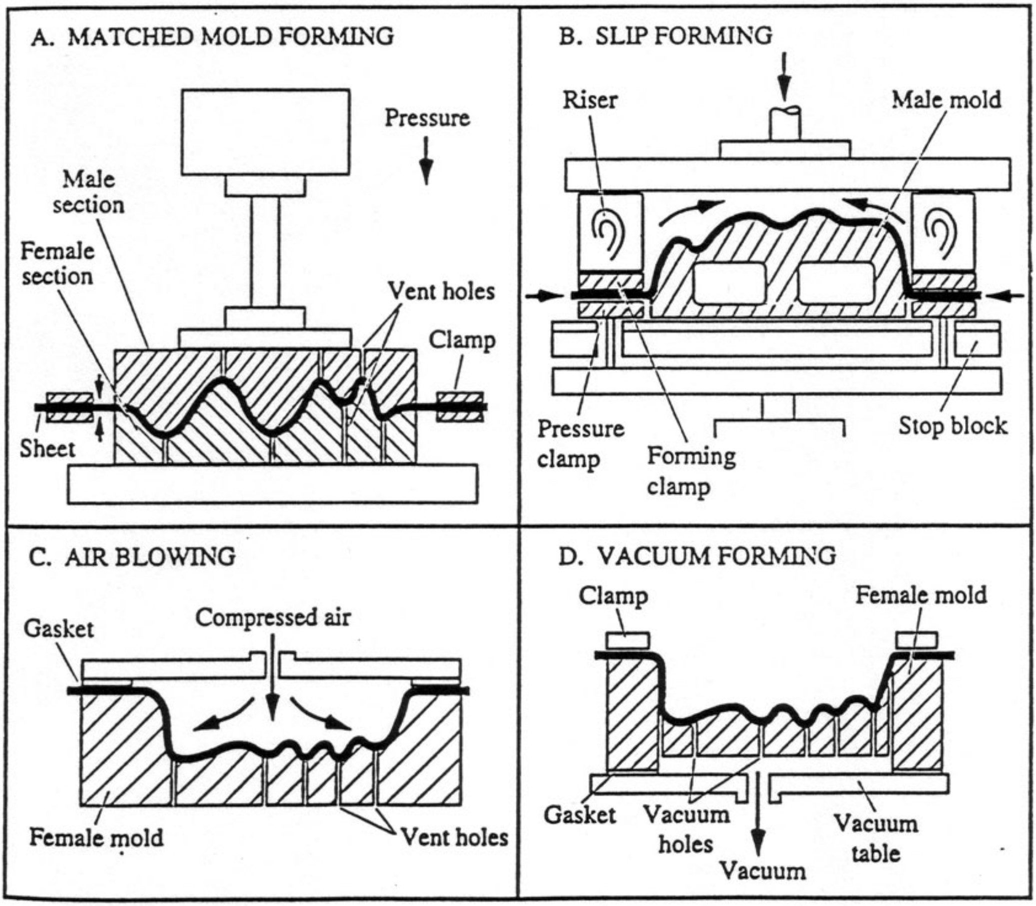


Fig. 10-2 Thermoforming classifications [2].

THERMOFORMING
VACUUM FORMING

$$\Delta P = 1\text{atm}.$$

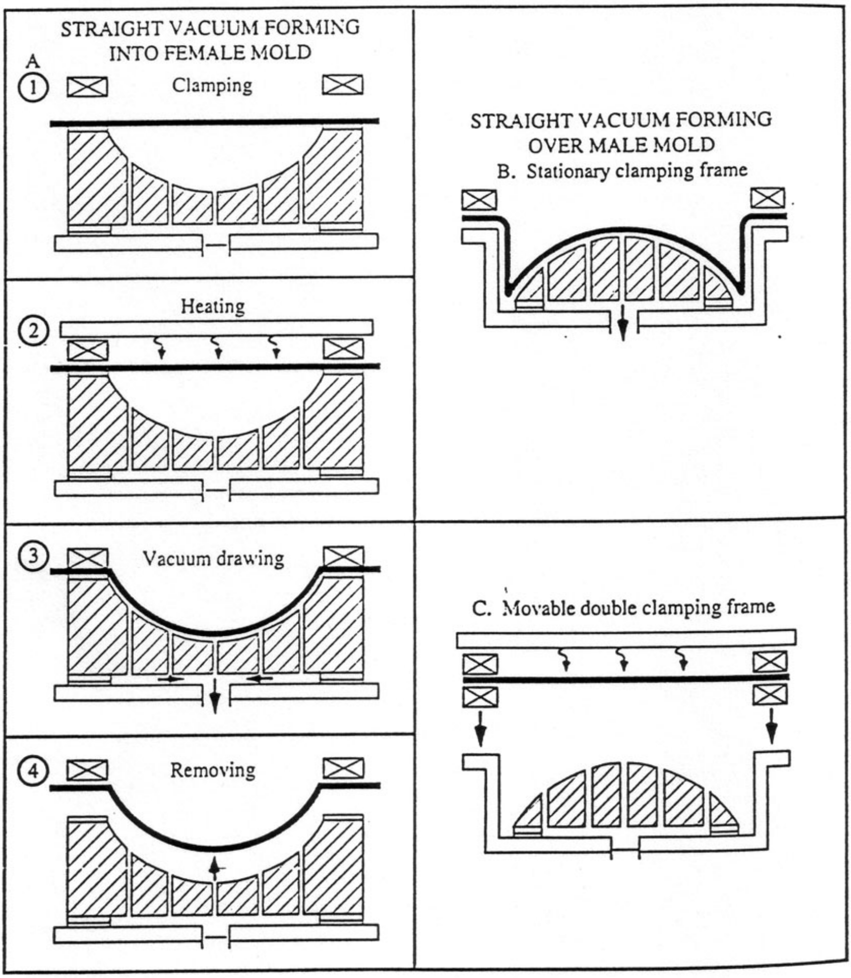
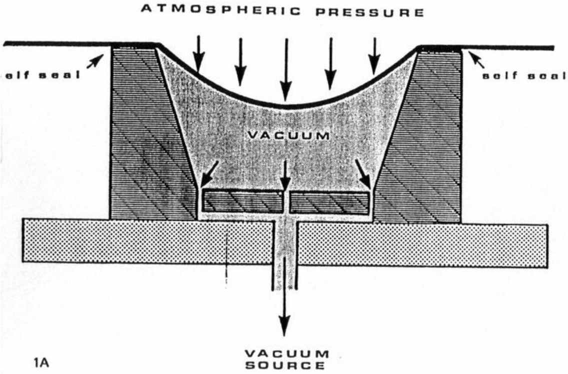


Fig. 10-8 Methods of straight vacuum forming [2].

THERMOFORMING VENTING

PROPER VENTING



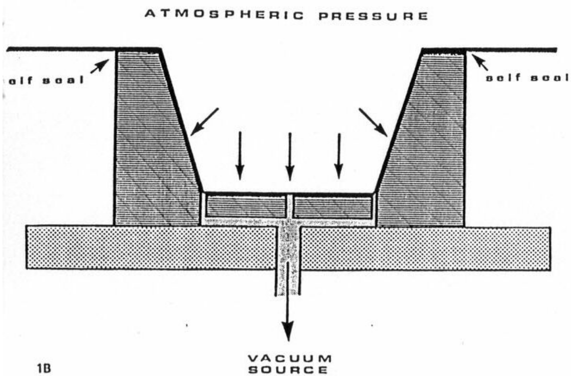
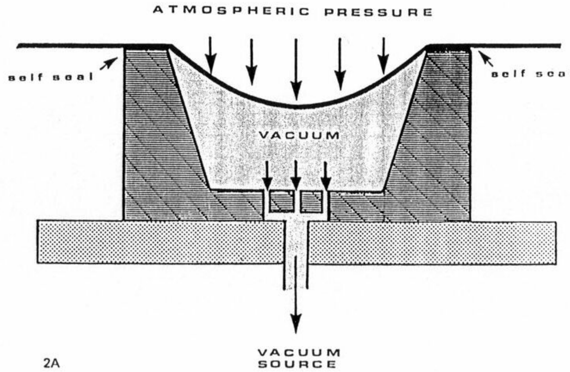


FIGURE 75 Mold with correct vacuum hole placement: (1A) forming begins satisfactorily; (1B) full detailed forming is made.

IMPROPER VENTING



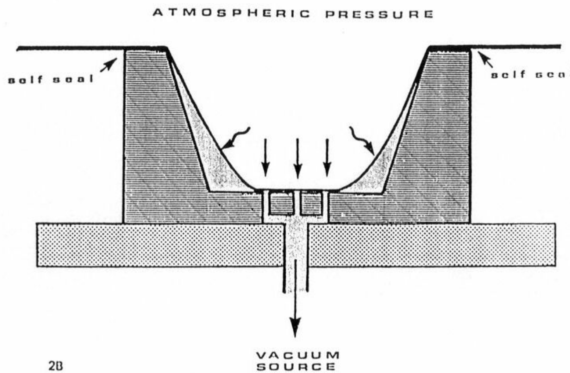


FIGURE 75 (continued) Mold with incorrect vacuum hole placement: (2A) forming will begin equally well; (2B) centrally located holes covered by forming plastic with some trapped air behind.

THERMOFORMING PRESSURE-ASSISTED

To exceed $\Delta P = 1$ atm. use pressurized air.

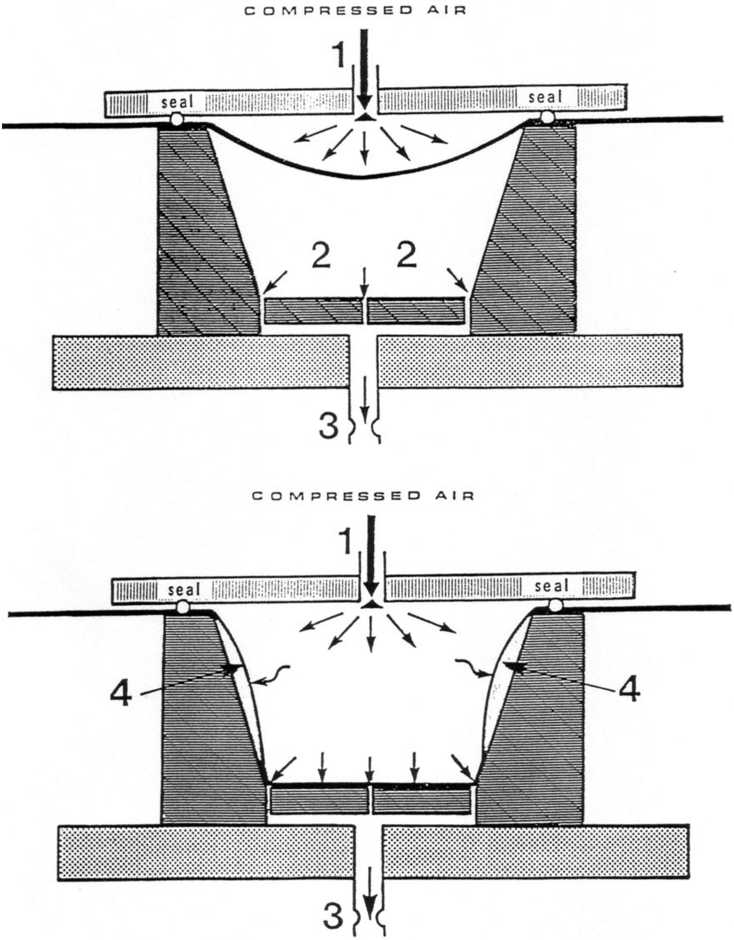


FIGURE 76 Overpressurized forming condition: (1) incoming pressure force; (2) undersized or small number of vent/vacuum holes; (3) restricted vent/vacuum channel; (4) trapped air pockets.

Avoid trapped air by using vacuum.