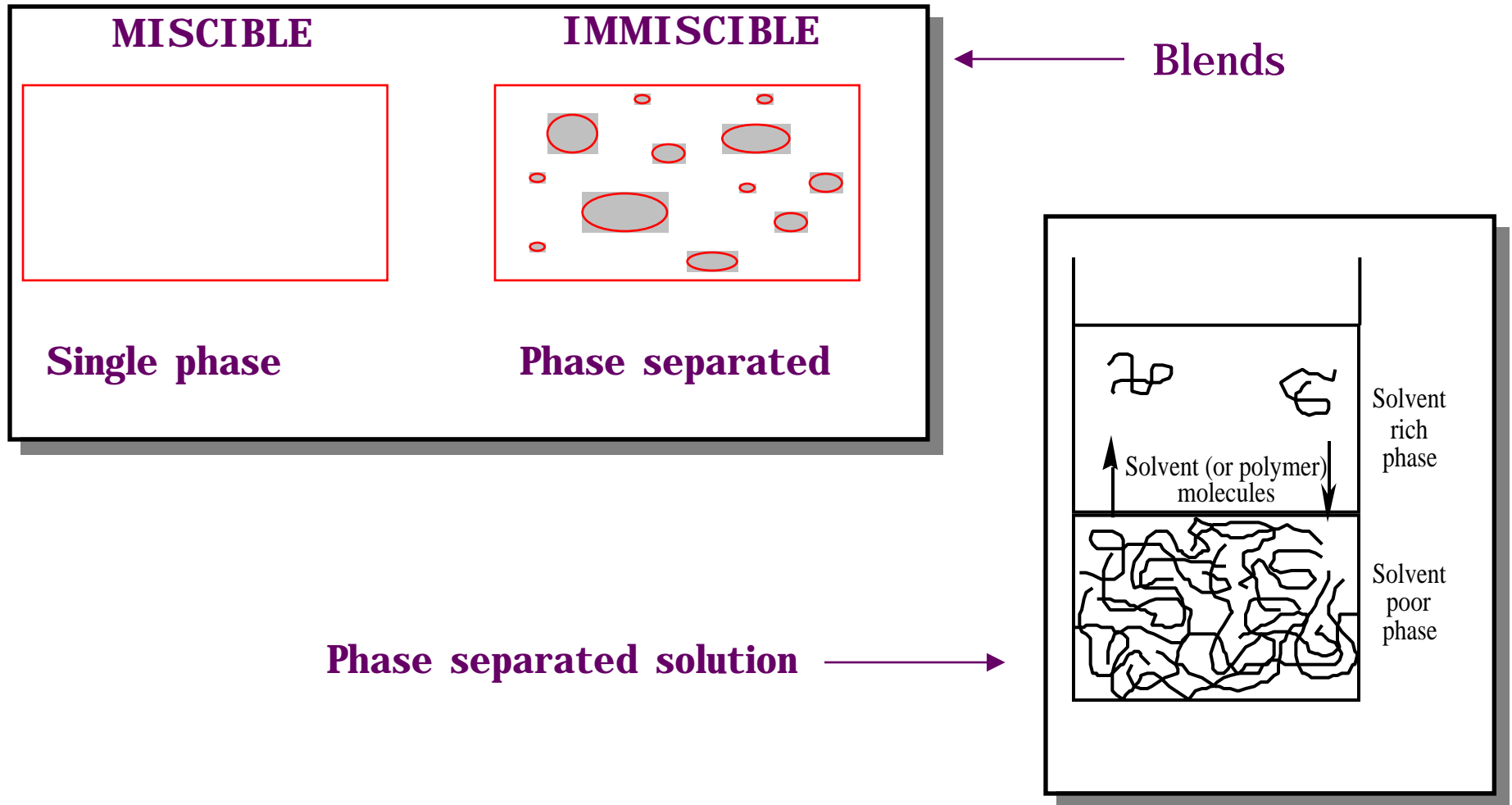


MIXING

- WILL A POLYMER DISSOLVE IN A GIVEN SOLVENT ?
- WILL A PARTICULAR POLYMER MIX WITH ANOTHER POLYMER ?

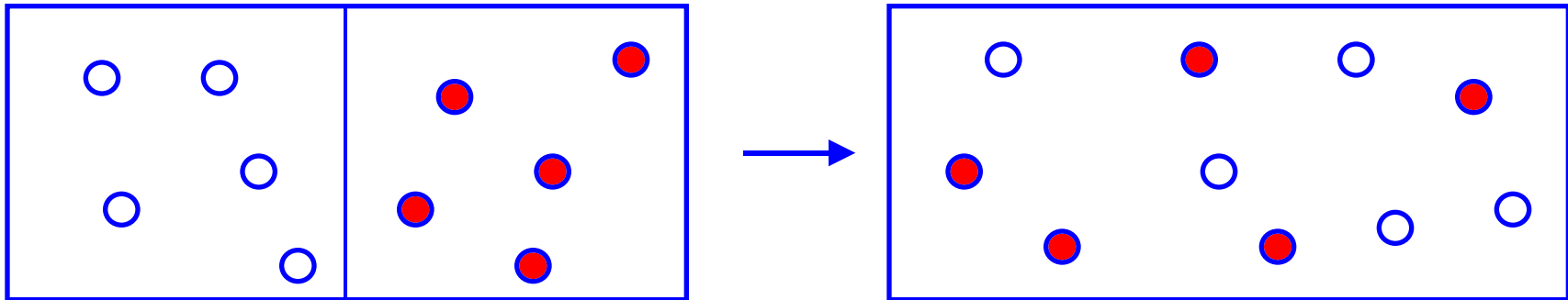
POLYMER SOLUTIONS AND BLENDS



CAN WE PREDICT PHASE BEHAVIOUR ?

THE SECOND LAW OF THERMODYNAMICS AND FREE ENERGY

For a process to occur spontaneously, S must increase; eg mixing



$$S_{\text{tot}} = S_{\text{sys}} + S_{\text{surr}}$$

But, if heat is released by the system

$$S_{\text{surr}} = - \frac{Q_{\text{sys}}}{T} = - \frac{H_{\text{sys}}}{T}$$

Define free energy

$$G = -T S_{\text{tot}} = H - T S$$

WHY BOTHER TO CALCULATE THE FREE ENERGY ?

- 1. BECAUSE IF WE DON'T THE GHOSTS
OF BOLTZMANN AND EHRENFEST WILL
COME BACK TO HAUNT US**
- 2. ONE CONDITION FOR FORMING A
SINGLE PHASE IS THAT THE FREE
ENERGY MUST BE NEGATIVE**
- 3. THE OTHER CONDITION IS THAT THE
SECOND DERIVATIVE OF THE FREE
ENERGY MUST BE POSITIVE**

THEORIES OF MIXING

$$G_m = H_m - T S_m$$

Assume the interactions between the molecules
Can be approximated by a mean field. This allows
The enthalpy and entropy to be treated separately

BOLTZMANN'S TOMB

Entropy is associated with the distribution of energy and matter in a system. This can be expressed formally in terms of the equation carved on Boltzmann's tomb

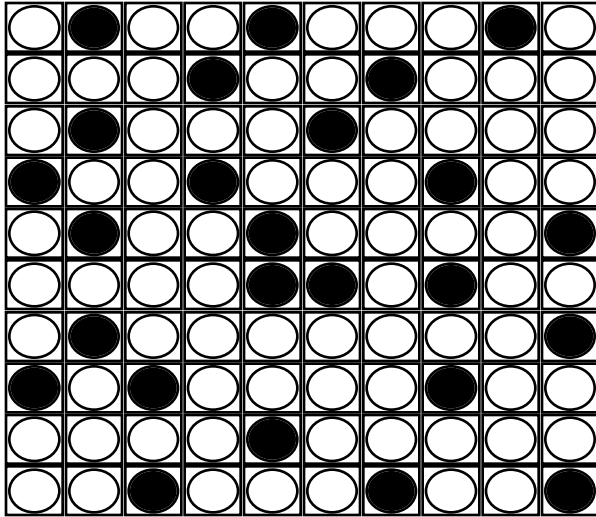
$$S = k \ln W$$

Which, today, is normally written

$$S = k \ln$$

Where Ω is the number of arrangements
Available to the system [At a given V, E, N]

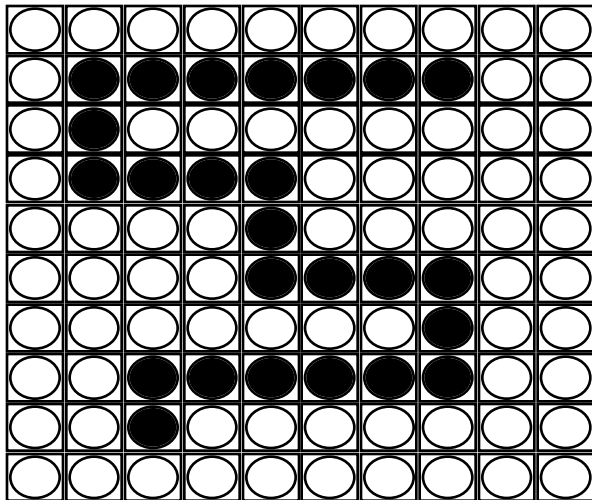
LATTICE MODELS - Summary



NUMBER OF CONFIGURATIONS

Equal- size small molecules

$$\frac{n_o !}{n_A ! n_B !}$$



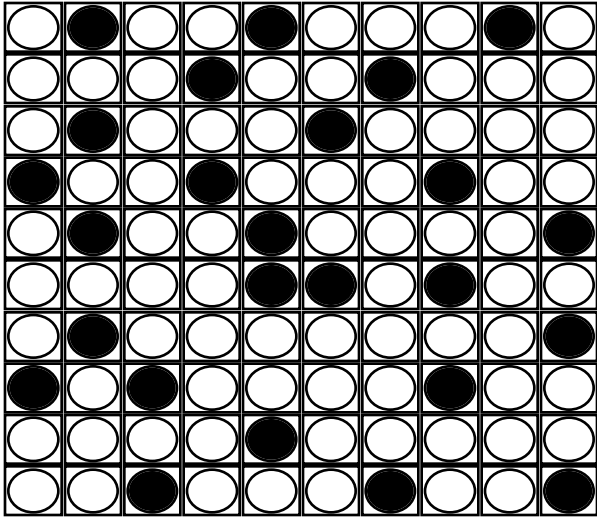
Flory model for polymer solutions

$$\frac{n_o !}{n_A ! N_B !} \left[\frac{N_B (m - 1)}{(z - 1)} \frac{1}{n_o^{N_B (m - 1)}} \right]$$

○ Solvent ● Polymer chain segment

How are these expresions obtained ?

REGULAR SOLUTION THEORY



NUMBER OF CONFIGURATIONS

$$= \frac{[N_A + N_B]!}{N_A! N_B!}$$

HENCE

$$-\frac{S_m}{R} = n_A \ln x_A + n_B \ln x_B$$

SOME NOTES CONCERNING THE ENTROPY OF MIXING

THE QUANTITIES N_A OR N_B CAN EITHER BE
THE NUMBER OF MOLECULES OF A AND B,
RESPECTIVELY, OR THE NUMBER OF MOLES

THE CONSTANT CHANGES DEPENDING ON CHOICE

MOLES	→	R (or RT)
MOLECULES	→	k (or kT)

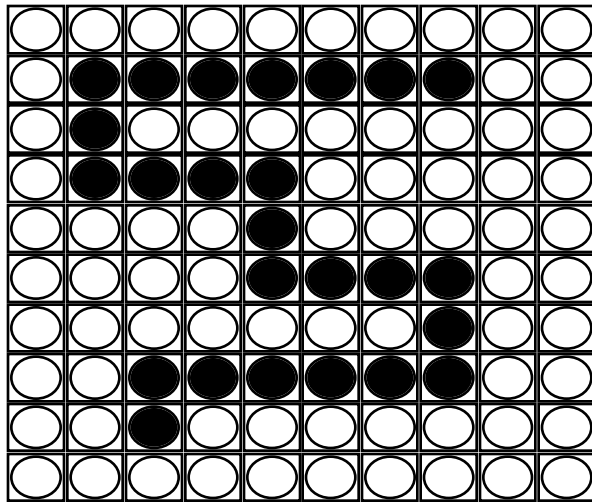
IN CALCULATING S_m WE USED STIRLING'S
APPROXIMATION

$$\ln(N!) = N \ln N - N$$

THE FINAL ANSWER IS WRITTEN IN TERMS OF
MOLE FRACTIONS X_A, X_B

$$X_A = \frac{N_A}{N_A + N_B} \quad X_B = \frac{N_B}{N_A + N_B}$$

POLYMER SOLUTIONS AND BLENDS



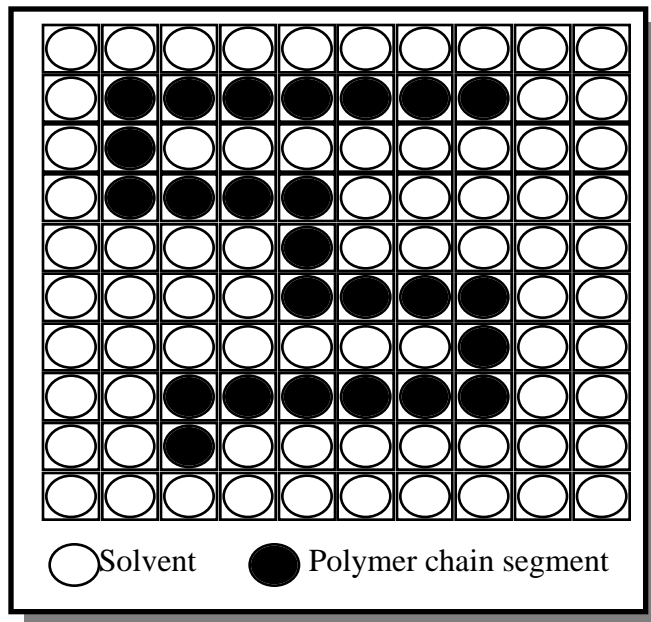
○ Solvent ● Polymer chain segment

COUNTING THE CONFIGURATIONS IS MORE DIFFICULT, BUT FLORY AND HUGGINS OBTAINED;

$$-\frac{S_m}{R} = n_A \ln \frac{A}{A+B} + n_B \ln \frac{B}{A+B}$$

DEFINITION OF VOLUME FRACTIONS

$$-\frac{S_m}{R} = n_s \ln s + n_p \ln p$$



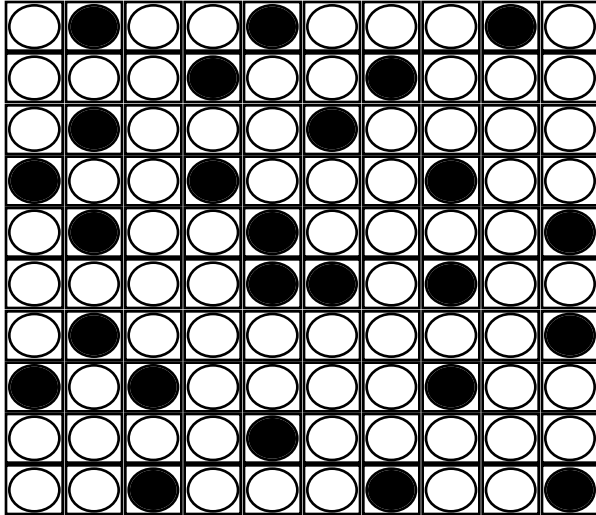
$$s = \frac{n_s}{n_s + Mn_p}$$

75 / 100

$$p = \frac{Mn_p}{n_s + Mn_p}$$

25/100

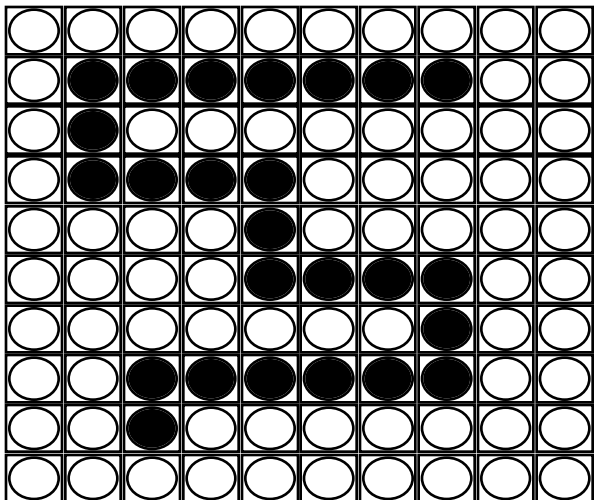
THE ENTROPY OF MIXING POLYMERS RELATIVE TO SMALL MOLECULES



$$\left[-\frac{S}{R} \right] = n_A \ln A + n_B \ln B$$

$$n_A = 75$$

$$n_B = 25$$



$$\left[-\frac{S}{R} \right] = n_A \ln A + n_B \ln B$$

$$n_A = 75$$

$$n_B = 1$$

○ Solvent ● Polymer chain segment

THE ENTHALPY OF MIXING

The enthalpy of the system depends upon the interactions
Between the molecules - we need an expression for the
Change in enthalpy on going from the pure components
To the mixture

There are two related approaches that are most commonly
Used

Hildebrand - solubility parameters

Flory - parameter

Both depend upon considering interactions
Between pairs of molecules and both assume
Dispersion and weak polar forces only

INTERACTIONS

DISPERSION FORCES

DIPOLE / DIPOLE
(freely rotating)

**HYDROGEN BONDS and
STRONG POLAR FORCES**
(charge transfer)

COULOMBIC
(eg ionomers)



Interaction energy

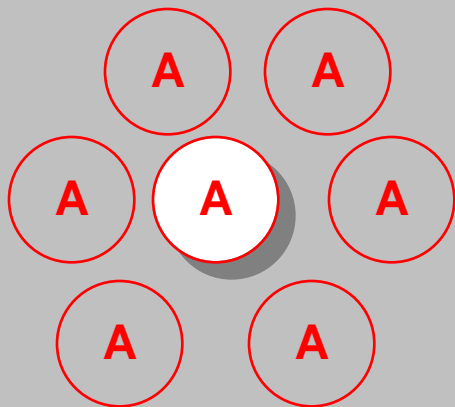
$$\sim \frac{1}{r^6}$$

**Interaction energy
complicated, but
short range**

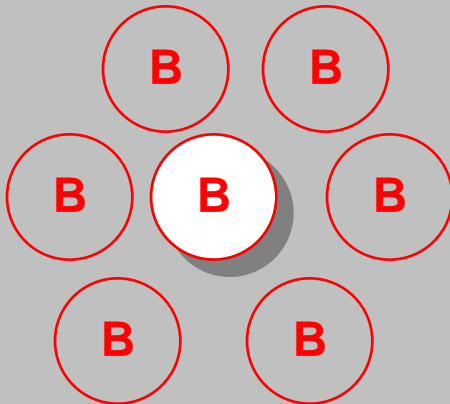
**Interaction energy
longer range
 $\sim 1/r$**

COHESIVE ENERGY DENSITY

For the pure components, define the cohesive energy per pair of molecules as;

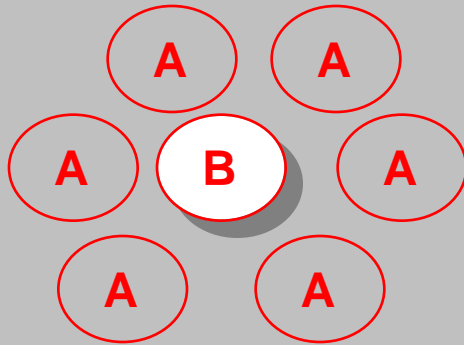


$$C_{AA} = \frac{E_{\text{vap}}}{V} - P.E.$$



$$C_{BB} = \frac{E_{\text{vap}}}{V} - P.E.$$

MIXTURES



Change in energy (per pair) related to

$$-[2C_{AB} - C_{AA} - C_{BB}]$$

Assume $C_{AB} = C_{AA}^{1/2} C_{BB}^{1/2}$

Obtain $[C_{AA} - 2C_{AA}^{1/2} C_{BB}^{1/2} + C_{BB}]$



$$[C_{AA}^{1/2} - C_{BB}^{1/2}]^2$$

The molar energy difference per pair on forming AB contacts from AA and BB contacts. Now multiply by the number of AB contacts in the mixture. But first, simplify !

SOLUBILITY PARAMETERS AND THE ENTHALPY OF MIXING

$$\delta_A = C_{AA}^{1/2}$$

$$\delta_B = C_{BB}^{1/2}$$

Define solubility parameters

$$(\delta_A - \delta_B)^2$$

The cohesive energy density change upon forming AB pairs from AA and BB pairs

Remember that this is per mole. To get the heat of mixing we must now multiply by the molar volume (to get an energy from the energy density), then the number of moles, then a factor that gives the number of AB contacts

$$H_m = (n_A + n_B) V_m \delta_A \delta_B (\delta_A - \delta_B)^2$$

FREE ENERGY OF MIXING- REGULAR SOLUTION THEORY

$$G_m = H_m - T S_m$$

$$\begin{aligned} \frac{G_m}{RT} = & n_A \ln X_A + n_B \ln X_B \\ & + (n_A + n_B) V_m A \left(\frac{1}{2} - \frac{1}{2} \right)^2 \end{aligned}$$