MECHANICAL AND RHEOLOGICAL PROPERTIES

MECHANICAL PROPERTIES OF SOLIDS

**Extension**

- Hooke's law: $\sigma = E\varepsilon$

**Shear**

- Hooke's law: $\tau_{xy} = G\gamma_{xy}$
MECHANICAL AND Rheological properties of fluids

RHEOLOGICAL PROPERTIES OF FLUIDS

Newtonian fluid

\[ \tau_{xy} = \eta \dot{\gamma} \]
OVERVIEW

POLYMERS TREATED AS SOLIDS
- Strength
- Stiffness
- Toughness

POLYMERS TREATED AS FLUIDS
- Viscosity of polymer melts
- Elastic properties of polymer melts

VISCOELASTIC PROPERTIES
- Creep
- Stress relaxation
STRESSES AND STRAINS
(or why you don’t fall through the floor)

HOW DOES AN INANIMATE OBJECT SUPPORT A LOAD?
In 1676 Hooke published "a decimate of the centesme of the inventions I intend to publish" (!!!)

Included "the true theory of elasticity or springiness"

As the extension, so the force

Questions: slope depends upon shape as well as the material being stretched - can we obtain a material property - stiffness?

How linear is the response of real materials?
**THERMODYNAMICS REVISTED**

\[ F = U - TS \]

For crystalline and glassy solids

\[ f \sim \left[ \frac{\partial U}{\partial l} \right]_{V,T} \]

For elastomer networks

\[ f \sim -T \left[ \frac{\partial S}{\partial l} \right]_{V,T} \]
MOLECULAR ORIGIN OF HOOKE'S LAW

\[
(dF)_{V,T} = (dE)_{V,T} - T(dS)_{V,T}
\]

ONE DIMENSIONAL CRYSTALLINE SOLID

Potential energy for a pair of atoms

For 3 atoms the potential field experienced by the middle atom is:

which can be summed to give the approximate form:

\[
f = K\Delta l + K\Delta l^2 + \ldots
\]

SINGLE POLMER CHAIN

Unstrained state

Strained state

\[
f = k\beta^2 R
\]
COMPARISON TO EXPERIMENT

POLYDIACETYLENE SINGLE CRYSTAL

Hooke's law obeyed up to deformations ~ 2%
Deviations beyond this could be due to defects as well as non linear terms in the force / extension equation

\[
\sigma = 2 \left( C_1 + C_2 / \lambda_1 \right) \lambda_1 - 1 / \lambda_1^2
\]

ELASTOMER NETWORK

Rubber networks obey hooke's law at low extensions, but not, perhaps surprisingly, at high strains (considering that deformation is reversible). The simplest theory of rubber elasticity gives

\[
f = N k T \left( \lambda_1 - 1 / \lambda_1^2 \right)
\]

but this is not that good. The semi-empirical Mooney - Rivlin equation provides a better fit

\[
\sigma = 2 \left( C_1 + C_2 / \lambda_1 \right) \lambda_1 - 1 / \lambda_1^2
\]
COMPARISON TO EXPERIMENT - POLYMERS

Once strains ~ 1% - 2% are reached, various types of deviations from ideal behavior are observed:

More on these later!
THE STIFFNESS OF MATERIALS

The stiffness of an object depends on its size and shape as well as the material from which it is constructed. We are interested in the inherent stiffness of materials and how they compare with one another.

First define stress = force / unit area

$$\sigma = \frac{F}{A}$$

This in itself is a useful concept.

**Example:**
- **Brick**: 3"x 4"; LOAD OF 200lbs
  - Stress = 16.67 lbs/sq.in.
- **Bridge**: 20' x 5'; LOCOMOTIVE ~ 100 TONS
  - Stress = \(\frac{100 \times 2,240}{5 \times 20 \times 144}\) = 15.6 lbs/sq in.
THE STIFFNESS OF MATERIALS

Now define strain:

$$\epsilon = \frac{l}{l_0}$$

Aircraft with strain of 1.6% in wings.
MODULUS OF DRAWN POLYETHYLENE
# Young's Modulus

Young \( \sim 1800 \) \( \frac{\text{Stress}}{\text{Strain}} \) = Constant

\[ \sigma = E \varepsilon \]

(After coming up with this, he got fired)

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>( E ) (lbs/sq in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber</td>
<td>( \sim 0.001 \times 10^6 )</td>
</tr>
<tr>
<td>Unreinforced Plastics</td>
<td>( \sim 0.2 \times 10^6 )</td>
</tr>
<tr>
<td>Wood</td>
<td>( \sim 2.0 \times 10^6 )</td>
</tr>
<tr>
<td>Concrete</td>
<td>( \sim 2.5 \times 10^6 )</td>
</tr>
<tr>
<td>Glass</td>
<td>( \sim 10.0 \times 10^6 )</td>
</tr>
<tr>
<td>Steel</td>
<td>( \sim 30.0 \times 10^6 )</td>
</tr>
<tr>
<td>Diamond</td>
<td>( \sim 170 \times 10^6 )</td>
</tr>
</tbody>
</table>
# Tensile Strength

<table>
<thead>
<tr>
<th>Material</th>
<th>TS psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel piano wire</td>
<td>450,000</td>
</tr>
<tr>
<td>High - tensile steel</td>
<td>225,000</td>
</tr>
<tr>
<td>Aluminium alloys</td>
<td>20,000 - 80,000</td>
</tr>
<tr>
<td>Titanium alloys</td>
<td>100,000 - 200,000</td>
</tr>
<tr>
<td>Wood (spruce), along grain</td>
<td>15,000</td>
</tr>
<tr>
<td>Wood (spruce), across grain</td>
<td>500</td>
</tr>
<tr>
<td>Ordinary glass</td>
<td>5,000 - 25,000</td>
</tr>
<tr>
<td>Ordinary brick</td>
<td>800</td>
</tr>
<tr>
<td>Ordinary cement</td>
<td>600</td>
</tr>
<tr>
<td>Nylon fiber</td>
<td>140,000</td>
</tr>
<tr>
<td>Kevlar 29 fiber</td>
<td>400,000</td>
</tr>
</tbody>
</table>

**Question Usually Asked**

Why is one material stronger than another?

**Question Should Be**

Why isn't any material as strong as it should be?
**HOW STRONG SHOULD MATERIALS BE?**

GRiffiths; calculated the theoretical strength of glass to be $\sim 1 \times 10^6$ p.s.i. measured values $\sim 25,000$ p.s.i.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>THEORETICAL STRENGTH</th>
<th>MEASURED STRENGTH</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STEEL</strong></td>
<td>$\sim 5 \times 10^6$ p.s.i</td>
<td>$\sim 400,000$ p.s.i (BEST)</td>
</tr>
<tr>
<td><strong>POLYETHYLENE FIBERS</strong></td>
<td>$\sim 25$ GPa</td>
<td>$\sim 0.35$ GPa (TENSILE DRAWING)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sim 4$ GPa (GEL SPUN)</td>
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