Residence Time Distribution

Without disturbing the steady-state flow, insert a dye marker uniformly across the cross sectional area of the input, at time $t = 0$.

What is the concentration of dye exiting the flow as a function of time?

Dye Concentration at Exit $f(t)$ (amount per unit time)

Residence Time $t$ (time dye takes to exit)

Mean Residence Time $\bar{t} = \int_0^{\infty} tf(t)dt$

**Example:** Newtonian flow in a circular pipe

$$P_1 > P_2 \quad v_r = v_\theta = 0 \quad v_z = \frac{\Delta P}{4\mu L} [R^2 - r^2]$$

Residence time depends on radial position because velocity depends on radial position.

$$t = \frac{L}{v_z} = \frac{4\mu L^2}{\Delta P [R^2 - r^2]}$$

Shortest residence time at centerline of pipe because the maximum velocity is there.

$$t_0 = \frac{L}{v_{max}} = \frac{4\mu L^2}{\Delta P R^2} \quad (t \text{ at } r = 0)$$
Residence Time Distribution

Range of residence times $t_0 \leq t \leq \infty$

(note: some dye is at the wall and never exits)

Distribution: $f(t)dt = \frac{2 \pi r v_z dr}{Q}$

Volumetric flow rate: $Q = \int_0^R 2 \pi r v_z dr = \frac{\pi \Delta P R^4}{8 \mu L}$

$$f(t)dt = \frac{\pi \Delta P}{2 \mu L} [R^2 - r^2] r dr$$

$$f(t)dt = 4 \left[ \frac{r}{R^2} - \frac{r^3}{R^4} \right] dr$$

$$t_0 \frac{t}{R} = \frac{4 \mu L^2}{\Delta P R^2} \left( \frac{R^2 - r^2}{R^2} \right) = 1 - (r/R)^2$$

$$(r/R)^2 = 1 - \frac{t_0}{t}$$

$$r = R \left( 1 - \frac{t_0}{t} \right)^{1/2}$$

$$dr = \frac{R}{2 t^2} \left( \frac{t_0 dt}{1 - \frac{t_0}{t}} \right)^{1/2}$$

$$f(t)dt = 4 \left[ \frac{(1 - \frac{t_0}{t})^{1/2}}{R} - \frac{(1 - \frac{t_0}{t})^{3/2}}{R} \right] \frac{R t_0 dt}{2 t^2 (1 - \frac{t_0}{t})^{1/2}}$$

$$= 2 \left[ 1 - \left( 1 - \frac{t_0}{t} \right) \right] \frac{t_0 dt}{t^2}$$

$$= \frac{2 t_0^2 dt}{t^3} \quad \text{for } t \geq t_0$$

$$= 0 \quad \text{for } t < t_0$$
Residence Time Distribution

Mean residence time:

\[ \bar{t} = \int_0^\infty t f(t) dt = \int_{t_0}^\infty \frac{2t_0^2 dt}{t^2} = -\frac{2t_0^2}{t} \bigg|_{t_0}^\infty \]

\[ \bar{t} = 2t_0 \]

Comparison of residence time distributions

For pipe flow: \( \bar{f}(t) = 4(t_0/t)^3 = (\bar{t}/t)^3/2 \)

Figure 1: Extruder flow has a narrower residence time distribution than pipe flow because the extruder has cross-channel flows and thus improved mixing.
TWIN-SCREW EXTRUSION

Figure 2: Different kinds of twin screw extruders: a) counter-rotating, intermeshing; b) co-rotating, intermeshing; c) counter-rotating, non-intermeshing; d) co-rotating, non-intermeshing.

Figure 3: Various leakage flows in the extruder.

Get better axial mixing with a twin-screw than with a single-screw extruder. Important for 2-phase blends.
Residence Time Distribution in Twin-Screw Extruders

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Twin screw extruders are finding increased usage in reacting and devolatilizing applications. Using self-screens profiles, the twin screws fulfill the requirement that there be no "dead" or "trimmed" zones. Agitated design must be chosen with care so that a reasonable balance can be obtained between forwarding rate, surface-generated rate, vapor paragogy, power, and axial mixing. Techniques have been developed for measuring residence time distributions and characterizing solid flow behavior. The method also permits direct determination of the holdup in milled barrel applications. Data on residence time distributions are presented for 4 in. diameter twin screw equipment with a variety of rotor configurations.


Figure 4: Pressure build up in a co-rotating twin-screw extruder.
THE BREAKER PLATE

Fig. 5.10 Example of a breaker plate assembly. 1 Inlet, 2 Outlet, 3 Screen pack, 4 Breaker plate

Fig. 5.6 Velocity profiles during the flow through a breaker plate. 1 Stagnation zone, 2 Inlet region, 3 Orifice, 4 Melt strand

Fig. 5.7 Breaker plate with conical boreholes
The multistrand die and slotted pulley keep hot strands from adhering to each other.
VARIANTS OF PELLETIZING

Fig. 5.1 Hot pelletizers [1], a) Pelletizer with a knife shaft in the center b) Pelletizer with an eccentric knife shaft,
1a and 1b – Knife, 2a and 2b – Knife holder, 3a and 3b – Knife shaft, 4a and 4b – Pelletizer die, 5a and 5b – Pellet collector housing, 6 – Torpedo

Fig. 5.2 Pelletizer die (geometry)

Fig. 5.3 Underwater pelletizer die [3]
Heat Transfer

\[ \rho C_p \frac{dT}{dt} = k \nabla^2 T \]  
(a diffusion equation)

\( \rho = \) density \hspace{1cm} \( C_p = \) specific heat \hspace{1cm} \( k = \) thermal conductivity

(anaologous to Fick’s Law \( \frac{dC}{dt} = D \frac{d^2C}{dx^2} \))

Thermal diffusivity \( \alpha \equiv \frac{k}{\rho C_p} \hspace{1cm} \text{(m}^2/\text{s})\)

\[ \frac{dT}{dt} = \alpha \nabla^2 T \]

Example: Cooling a cylindrical extrudate

\[ T = T(r, t) \]

\[ \frac{\alpha t}{R^2} \] is dimensionless; \( T \) is the temperature at \( r \); \( T_0 \) is the extrudate melt temperature; \( T_1 \) is the bath temperature

Fig. 9.6 Temperature profiles for unsteady state heat conduction in an infinite cylinder: \( T(r, 0) = T_0, T(R, t) = T_1 \). (Reprinted with permission from H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, New York, 1973.)
Heat Transfer

**Example:** Pelletizing polystyrene

Thermal diffusivity \( \alpha = 7 \times 10^{-4} \text{cm}^2/\text{s} \)

Extrudate \( T_0 = 200^\circ \text{C} \)     Bath \( T_1 = 40^\circ \text{C} \)

When will the center of a 1mm radius extrudate reach \( T_g = 100^\circ \text{C} \)?

Center means \( r = 0 \)

\[
\frac{T - T_0}{T_1 - T_0} = \frac{-100}{-150} = \frac{2}{3}
\]

![Graph](image)

**Fig. 9.6** Temperature profiles for unsteady state heat conduction in an infinite cylinder: \( T(r, 0) = T_0, T(R, t) = T_1 \). (Reprinted with permission from H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, New York, 1973.)

From graph, \( \frac{\alpha t}{R^2} \approx 0.3 \)

\[ t = \frac{0.3R^2}{\alpha} = \frac{0.3(0.1\text{cm})^2}{7 \times 10^{-4}\text{cm}^2/\text{s}} = 4\text{s} \]