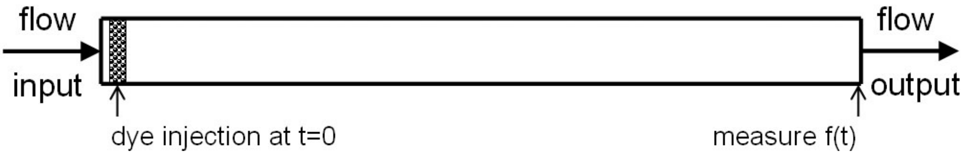
Residence Time Distribution

Without disturbing the steady-state flow, insert a dye marker uniformly across the cross sectional area of the input, at time $t = 0$.



What is the concentration of dye exiting the flow as a function of time?

Dye Concentration at Exit $f(t)$ (amount per unit time)

Residence Time t (time dye takes to exit)

Mean Residence Time $\bar{t} = \int_0^\infty t f(t) dt$

Example: Newtonian flow in a circular pipe

$$P_1 > P_2 \qquad v_r = v_\theta = 0 \qquad v_z = \frac{\Delta P}{4\mu L}\left[R^2 - r^2\right]$$

Residence time depends on radial position because velocity depends on radial position.

$$t = \frac{L}{v_z} = \frac{4\mu L^2}{\Delta P[R^2 - r^2]}$$

Shortest residence time at centerline of pipe because the maximum velocity is there.

$$t_0 = \frac{L}{v_{max}} = \frac{4\mu L^2}{\Delta P R^2} \qquad (t \text{ at } r = 0)$$

Residence Time Distribution

Range of residence times $t_0 \le t \le \infty$
(note: some dye is at the wall and never exits)

$$\text{Distribution:} \qquad f(t)dt = \frac{2\pi r v_z dr}{Q}$$

$$\text{Volumetric flow rate:} \qquad Q = \int_0^R 2\pi r v_z dr = \frac{\pi \Delta P R^4}{8\mu L}$$

$$f(t)dt = \frac{\frac{\pi\Delta P}{2\mu L}[R^2 - r^2] r dr}{\frac{\pi \Delta P R^4}{8\mu L}}$$

$$f(t)dt = 4\left[\frac{r}{R^2} - \frac{r^3}{R^4}\right] dr$$

$$\frac{t_0}{t} = \frac{4\mu L^2/(\Delta P R^2)}{4\mu L^2/(\Delta P[R^2 - r^2])} = \frac{R^2 - r^2}{R^2} = 1 - (r/R)^2$$

$$(r/R)^2 = 1 - \frac{t_0}{t}$$

$$r = R\left(1 - \frac{t_0}{t}\right)^{1/2}$$

$$dr = \frac{R}{2t^2}\frac{t_0 dt}{\left(1 - \frac{t_0}{t}\right)^{1/2}}$$

$$\begin{aligned} f(t)dt &= 4\left[\frac{\left(1-\frac{t_0}{t}\right)^{1/2}}{R} - \frac{\left(1-\frac{t_0}{t}\right)^{3/2}}{R}\right]\frac{R t_0 dt}{2t^2\left(1-\frac{t_0}{t}\right)^{1/2}} \\ &= 2\left[1 - \left(1 - \frac{t_0}{t}\right)\right]\frac{t_0 dt}{t^2} \\ &= \frac{2t_0^2 dt}{t^3} \qquad \text{for } t \ge t_0 \\ &= 0 \qquad \text{for } t < t_0 \end{aligned}$$

Residence Time Distribution

Mean residence time:

$$\bar{t} = \int_0^\infty t f(t) dt = \int_{t_0}^\infty \frac{2t_0^2 dt}{t^2} = -\frac{2t_0^2}{t}\bigg|_{t_0}^\infty$$

$$\bar{t} = 2t_0$$

Comparison of residence time distributions

For pipe flow: $\bar{t}f(t) = 4(t_0/t)^3 = (\bar{t}/t)^3/2$

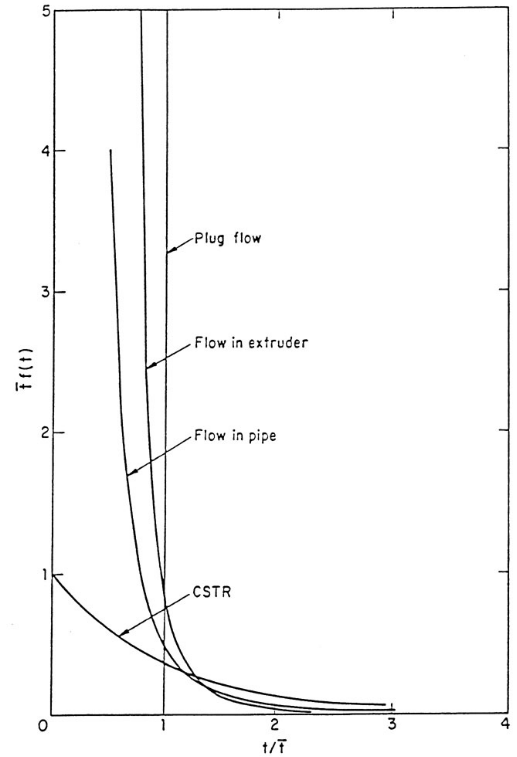


Figure 7.6 The residence time distribution function $t_f(t)$ versus reduced time $t/\bar{t}$ for flow in an extruder, compared to (a) plug flow (where all fluid particles have equal residence time), (b) isothermal flow of Newtonian fluid in cylindrical pipes, and (c) continuous stirred tank reactor. The basis of comparison was the equal mean residence time, $\bar{t}$, in all cases.

Figure 1: Extruder flow has a narrower residence time distribution than pipe flow because the extruder has cross-channel flows and thus improved mixing.

TWIN-SCREW EXTRUSION

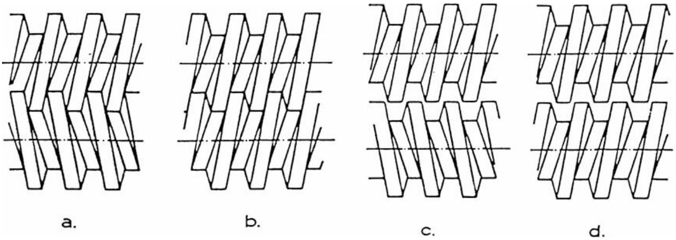


Figure 2: Different kinds of twin screw extruders: a) counter-rotating, intermeshing; b) co-rotating, intermeshing; c) counter-rotating, non-intermeshing; d) co-rotating, non-intermeshing.

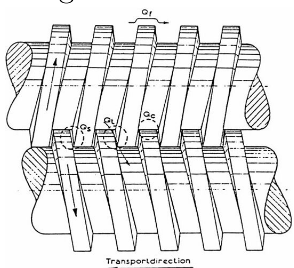


Figure 3: Various leakage flows in the extruder.

Get better axial mixing with a twin-screw than with a single-screw extruder. Important for 2-phase blends.

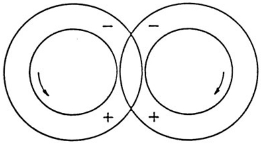


Fig. III.10

Pressure build up in a counter-rotating extruder.

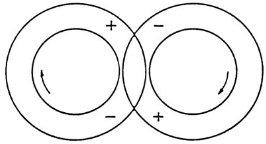


Fig. III.11

Pressure build up in a co-rotating extruder.

Residence Time Distribution

Residence Time Distribution in Twin-Screw Extruders

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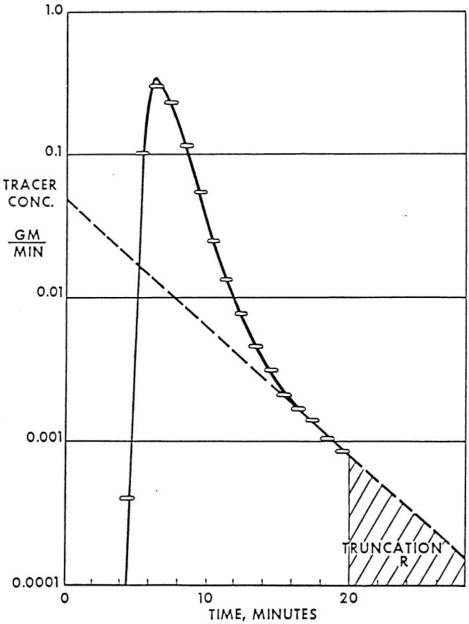
Saginaw, Michigan

Twin screw extruders are finding increased usage in reacting and devolatilizing applications. Using self-wiping profiles, the twin screws fulfill the requirement that there be no "dead" or "unmixed" zones. Agitator design must be chosen with care so that a reasonable balance can be obtained between forwarding rate, surface-generation rate, vapor passageway, power, and axial mixing. Techniques have been developed for measuring residence time distributions and characterizing axial flow behavior. The method also permits direct determination of the holdup in starved barrel applications. Data on residence time distribution are presented for 4-in. diameter twin screw equipment with a variety of rotor configurations.

Polym. Eng. Sci. 15, 437 (1975).



gure 4: Pressure build up in a co-rotating twin-screw extruder.

THE BREAKER PLATE

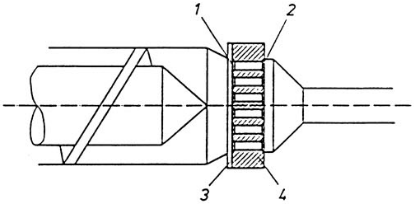


Fig. 5.10 Example of a breaker plate assembly. *1* Inlet, *2* Outlet, *3* Screen pack, *4* Breaker plate

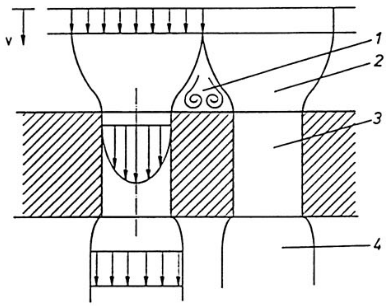


Fig. 5.6 Velocity profiles during the flow through a breaker plate. *1* Stagnation zone, *2* Inlet region, *3* Orifice, *4* Melt strand

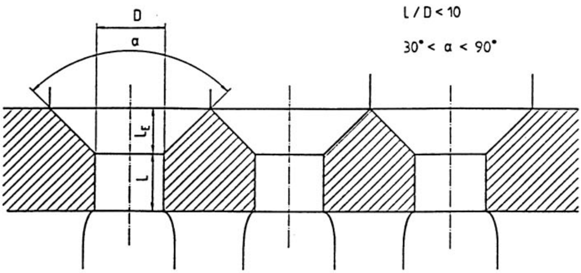
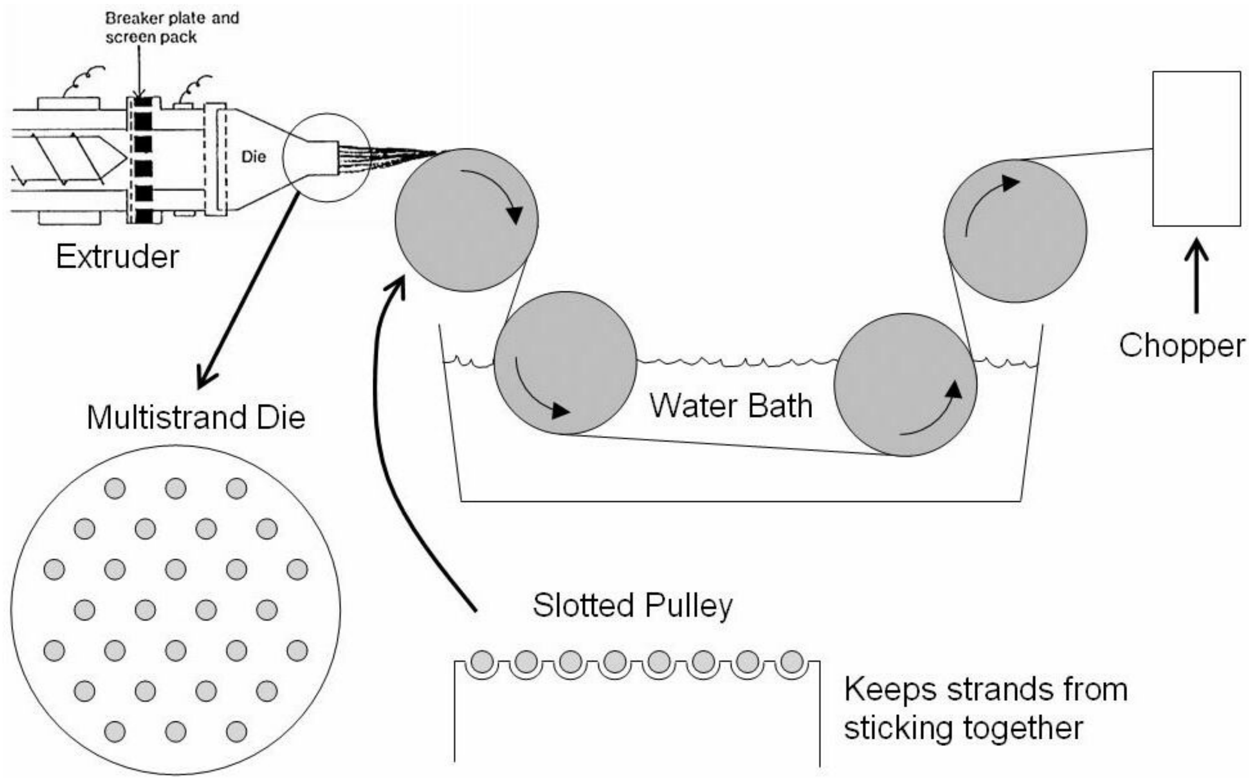


Fig. 5.7 Breaker plate with conical boreholes

PELLETIZING



The multistrand die and slotted pulley keep hot strands from adhering to each other.

VARIANTS OF PELLETIZING

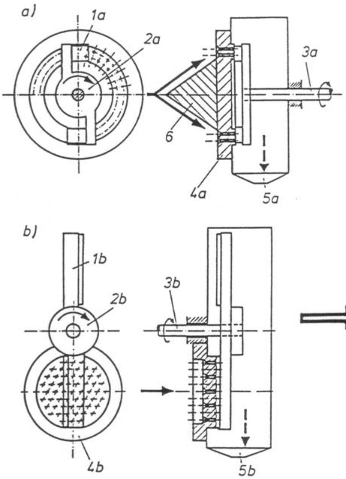


Fig. 5.1 Hot pelletizers [1], a) Pelletizer with a knife shaft in the center b) Pelletizer with an excentric knife shaft,
1 a and *1 b* – Knife, *2 a* and *2 b* – Knife holder, *3 a* and *3 b* – Knife shaft, *4 a* and *4 b* – Pelletizer die, *5 a* and *5 b* – Pellet collector housing, *6* – Torpedo

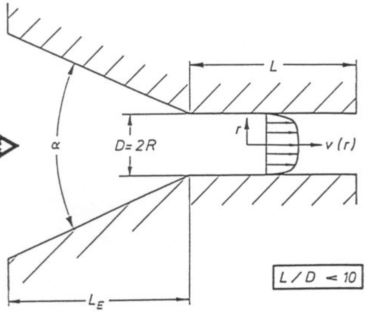


Fig. 5.2 Pelletizer die (geometry)

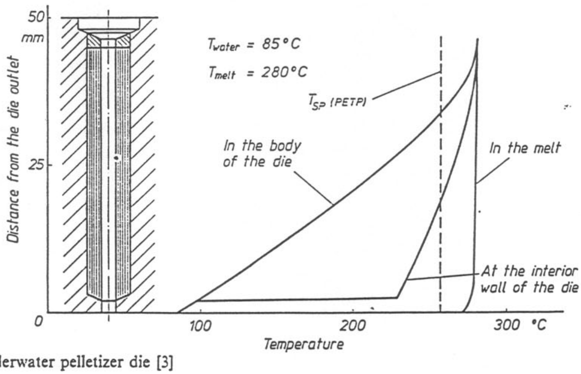


Fig. 5.3 Underwater pelletizer die [3]

Heat Transfer

$$\rho C_p \frac{dT}{dt} = k\nabla^2 T \qquad \text{(a diffusion equation)}$$

$\rho =$ density $\quad C_p =$ specific heat $\quad k =$ thermal conductivity

(analogous to Fick's Law $\frac{dC}{dt} = D\frac{d^2C}{dx^2}$)

Thermal diffusivity $\alpha \equiv \frac{k}{\rho C_p}$ (m^2/s)

$$\frac{dT}{dt} = \alpha \nabla^2 T$$

Example: Cooling a cylindrical extrudate

$$T = T(r, t)$$

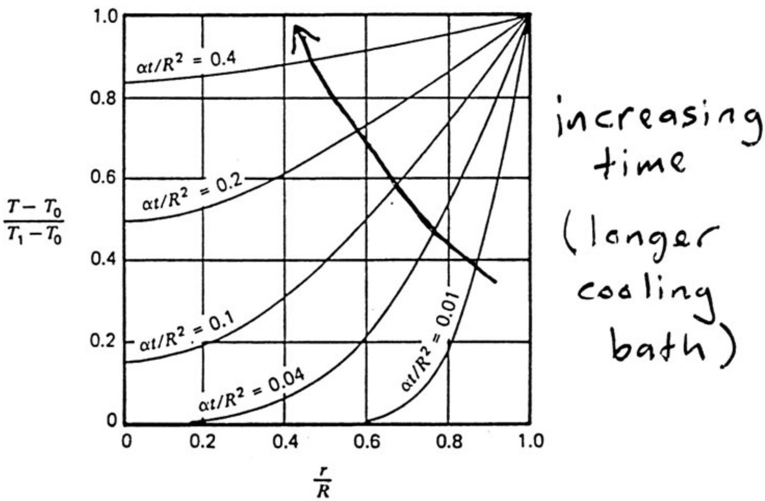


Fig. 9.6 Temperature profiles for unsteady state heat conduction in an infinite cylinder: $T(r, 0) = T_0$, $T(R, t) = T_1$. (Reprinted with permission from H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, New York, 1973.)

Where $\alpha t/R^2$ is dimensionless; T is the temperature at r;
T_0 is the extrudate melt temperature; T_1 is the bath temperature

Heat Transfer

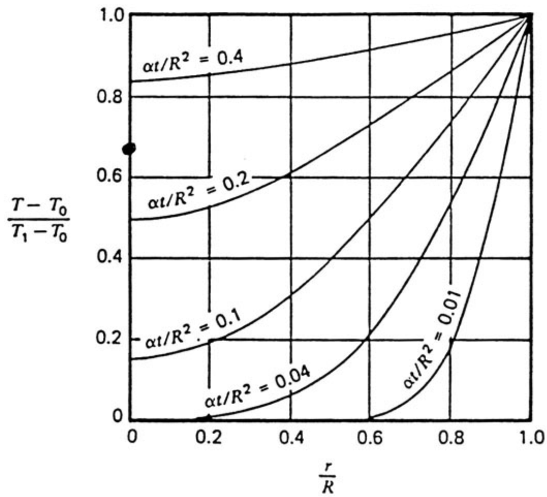
Example: Pelletizing polystyrene

Thermal diffusivity $\alpha = 7 \times 10^{-4}\text{cm}^2/\text{s}$

Extrudate $T_0 = 200°\text{C}$ Bath $T_1 = 40°\text{C}$

When will the center of a 1mm radius extrudate reach $T_g = 100°\text{C}$?
Center means $r = 0$

$$\frac{T - T_0}{T_1 - T_0} = \frac{-100}{-150} = \frac{2}{3}$$



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$$\text{From graph, } \frac{\alpha t}{R^2} \cong 0.3$$

$$t = \frac{0.3R^2}{\alpha} = \frac{0.3(0.1\text{cm})^2}{7 \times 10^{-4}\text{cm}^2/\text{s}} = 4\text{s}$$