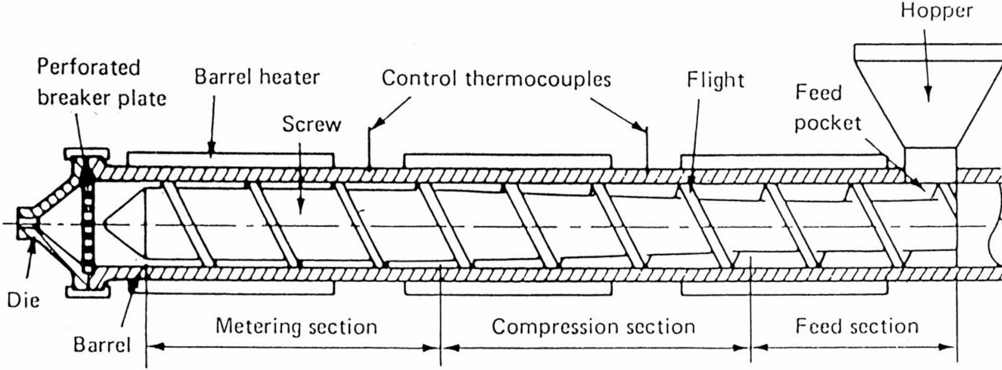
Extrusion



Extruders are sized by their barrel diameter D.

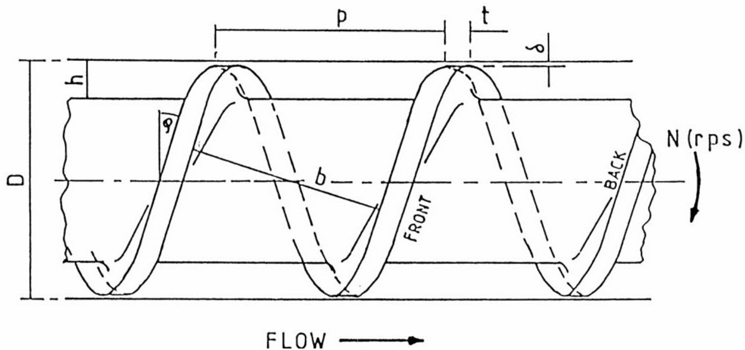


Figure 1: An extruder has two roles: Pumping & Mixing.

Extrusion

Unwind helical screw into a flat cartesian coordinate system.

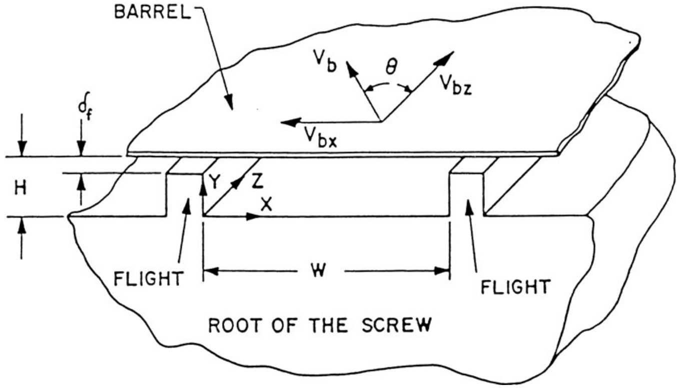


Figure 2: The extruder has the screw turning in a fixed barrel.

Choose coordinate system that moves with the screw. Then effectively have the barrel moving with velocity $\vec{v}_b$.

$$\vec{v}_b = -v_b \sin\theta \vec{i} + v_b \cos\theta \vec{k} \qquad v_b = |\vec{v}_b|$$

Time independent

$$v_y = 0 \qquad v_x = v_x(x, y) \qquad v_z = v_z(x, y)$$

Continuity

$$\frac{dv_x}{dx} = 0 \qquad \Rightarrow \qquad v_x = v_x(y)$$

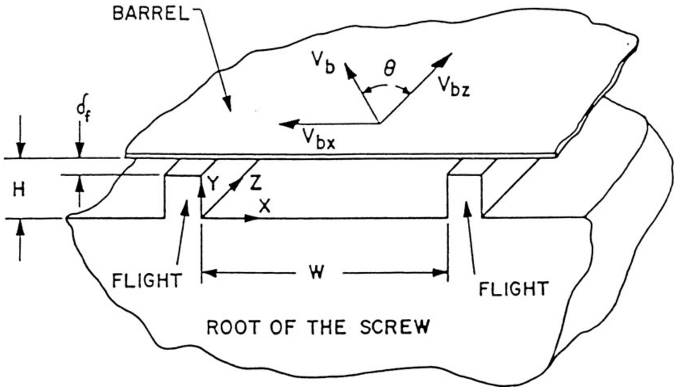
N.-S.:

$$\frac{dP}{dx} = \mu \frac{d^2 v_x}{dy^2} \qquad \frac{dP}{dy} = 0$$

$$\frac{dP}{dz} = \mu \left(\frac{d^2 v_z}{dx^2} + \frac{d^2 v_z}{dy^2} \right)$$

B.C. at $y = 0$, $\vec{v} = 0$, $v_x = v_z = 0$
at $y = H$, $\vec{v} = \vec{v}_b$, $v_x = -v_b \sin\theta$, $v_z = v_b \cos\theta$

Extrusion



$P = P(x, z)$

$$\frac{dP}{dx} = \mu \frac{d^2 v_x}{dy^2}$$

$$f(x, z) = h(y) \qquad \therefore \text{(both constant)}$$

There is no obvious B.C. for pressure. For now we just keep dP/dx and remember that it is constant!

$$\frac{d^2 v_x}{dy^2} = \frac{1}{\mu} \frac{dP}{dx} \qquad \text{a constant}$$

$$\text{Integrate twice:} \qquad v_x = \frac{1}{2\mu} \frac{dP}{dx} y^2 + C_1 y + C_2$$

$$v_x(0) = 0 \quad \Rightarrow \quad C_2 = 0$$

$$v_x(H) = -v_b \sin\theta = \frac{1}{2\mu} \frac{dP}{dx} H^2 + C_1 H$$

$$C_1 = -\frac{v_b}{H} \sin\theta - \frac{H}{2\mu} \frac{dP}{dx}$$

Extrusion

$$v_x = \frac{1}{2\mu}\frac{dP}{dx}y^2 - \left(\frac{v_b}{H}\sin\theta + \frac{H}{2_\mu}\frac{dP}{dx}\right)y$$

The vertical flights prevent any net flow in the x-direction.

Net flow in the x-direction is zero

$$\int_0^H v_x dy = 0$$

$$\left[\frac{1}{6\mu}\frac{dP}{dx}y^3 - \left(\frac{v_b}{H}\sin\theta + \frac{H}{2\mu}\frac{dP}{dx}\right)\frac{y^2}{2}\right]_0^H = 0$$

$$\frac{1}{6\mu}\frac{dP}{dx}H^3 - \frac{v_b H}{2}\sin\theta - \frac{H^3}{4\mu}\frac{dP}{dx} = 0$$

$$\frac{H^3}{12\mu}\frac{dP}{dx} = -\frac{v_b H}{2}\sin\theta$$

$$\frac{dP}{dx} = -\frac{6\mu}{H^2}v_b\sin\theta$$

Put into v_x:

$$v_x = -\frac{3y^2}{H^2}v_b\sin\theta - \frac{y}{H}v_b\sin\theta + \frac{3y}{H}v_b\sin\theta$$

$$v_x - v_b\sin\theta\frac{y}{H}\left[2 - 3\frac{y}{h}\right]$$

Extrusion

$$v_x = v_b \sin\theta \frac{y}{H}\left[2 - 3\frac{y}{h}\right]$$

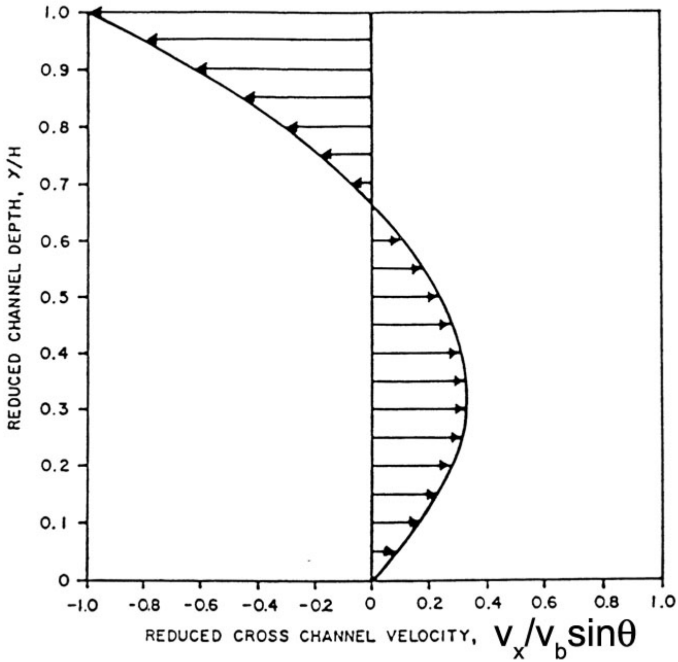


Figure 3: The extruder has the screw turning in a fixed barrel.

v_x is a **universal function** of $v_b \sin\theta$ and y/H.

independent of viscosity!

at $\quad y = \frac{2}{3}H, \qquad v_x = 0 \qquad$ Observed in experiment

Extrusion

$$\frac{dP}{dz} = \mu \left(\frac{d^2 v_z}{dx^2} + \frac{d^2 v_z}{dy^2} \right) \qquad v_z = v_z(x, y)$$

$$f(z) = h(x, y) \qquad \therefore \text{ both constant}$$

No B.C. for pressure, but dP/dz is constant.

$$\frac{d^2 v_z}{dx^2} + \frac{d^2 v_z}{dy^2} = \frac{1}{\mu} \frac{dP}{dz} \qquad \text{(a constant)}$$

$$\begin{aligned} \text{B.C.} \quad v_z(x, 0) &= 0 \qquad v_z(x, H) = v_b \cos\theta \\ v_z(0, y) &= v_z(w, y) = 0 \end{aligned}$$

Solution is more complicated: $\qquad v_{bz} \equiv v_b \cos\theta$

$$\begin{aligned} \frac{v_z}{v_{bz}} = {} & \frac{4}{\pi} \sum_{i=1,3,5}^{\infty} \frac{\sin h(i\pi y/W)}{i \sin h(i\pi H/W)} \sin\left(\frac{i\pi x}{W} \right) \\ & - \left(\frac{1}{2\mu} \frac{dP}{dz} \frac{H^2}{v_{bz}} \right) \left[\left(\frac{y}{H} \right)^2 - \frac{y}{H} + \frac{8}{\pi^3} \sum_{i=1,3,5}^{\infty} \frac{\cos h \left[i\pi \frac{W}{H} \left(\frac{x}{w} - \frac{1}{2} \right) \right]}{i^3 \cos h \left(\frac{i\pi W}{2H} \right)} \sin\left(\frac{i\pi y}{H} \right) \right] \end{aligned}$$

$$\text{i.e.} \qquad \frac{v_z}{v_{bz}} = \text{Drag flow} - \text{Pressure-driven backflow}$$

Drag flow term is a Couette flow (it comes from the moving boundary)

Pressure-driven backflow is a Poiseuille Flow

The origin of the pressure is due to some flow restriction at the outlet of the extruder (such as a die).

Extrusion

Volumetric flow rate $$Q = n\int_0^H \int_0^W v_z dx dy$$

$n =$ number of parallel channels

$$Q = Q_D + Q_P$$

$$Q_D = n v_b \cos\theta W H F_D(W/h)$$

$$Q_P = -n\frac{WH^3}{\mu}\left(\frac{dP}{dz}\right) F_p(W/H)$$

$$F_D = \frac{8W}{\pi^3 H}\sum_{iodd}\frac{1}{i^3}\tanh\left(\frac{i\pi H}{2W}\right)$$

$$F_P = \frac{1}{12} - \frac{16H}{\pi^5 W}\sum_{iodd}\frac{1}{i^5}\tanh\left(\frac{i\pi H}{2W}\right)$$

$$\lim_{H/W\to 0} F_D = 1/2 \qquad \lim_{H/W\to 0} F_P = 1/12$$

In the **shallow channel limit** ($H/W \ll 1$):

$$-\frac{Q_P}{Q_D} = \frac{H^2}{6\mu v_b \cos\theta}\frac{dP}{dz}$$

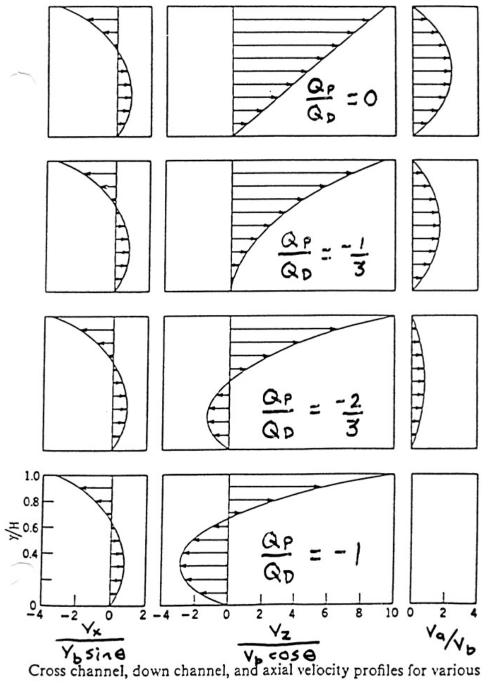
In the middle of the channel ($x = W/2$) can neglect the flights of the screw.

$$v_z = v_b \cos\theta\left[\frac{y}{h} + 3\frac{y}{H}\frac{Q_P}{Q_D}\left(1 - \frac{y}{H}\right)\right]$$

Extrusion

Velocity along the axial direction of the extruder is a vector sum of v_x and v_z.

$$v_a = v_x \cos\theta + v_z \sin\theta$$



Cross channel, down channel, and axial velocity profiles for various Q_p/Q_c values.

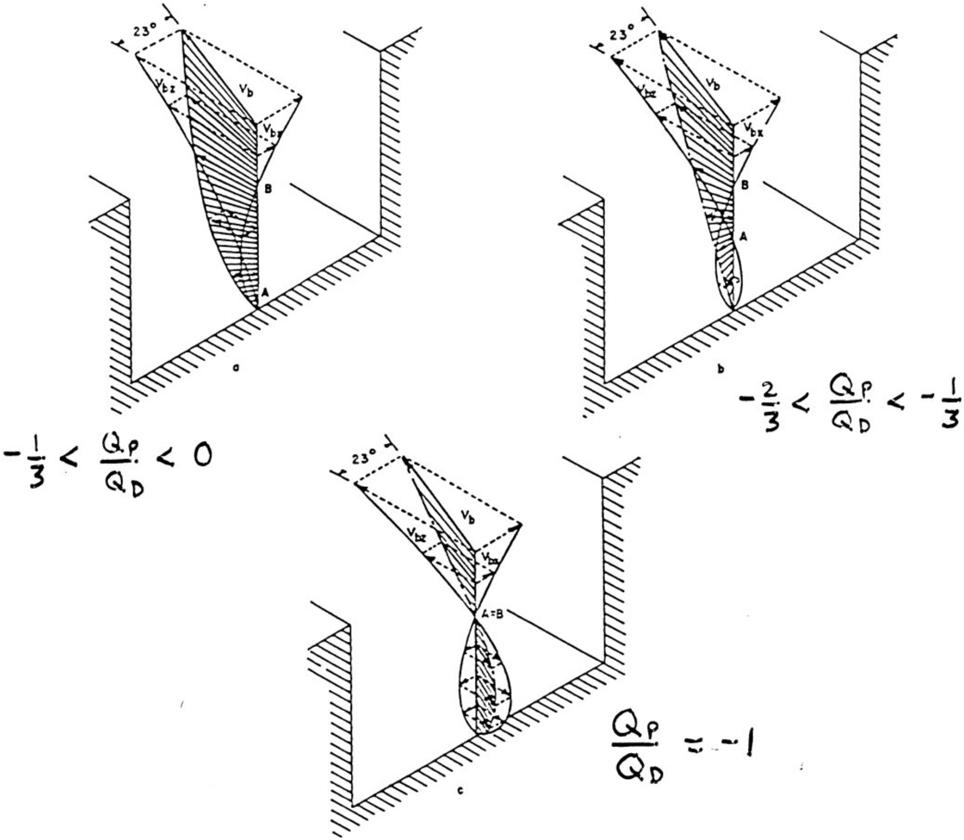
No flow restriction (pure drag flow) Optimum Pumping

$v_z = v_b \cos\theta (y/H)^2$

$v_z = 0$ at $y = 0$ and at $y = H/2$

Plugged extruder outlet (zero net flow). Optimum mixing.

3D MID-CANNEL VELOCITY PROFILE



Increase flow restriction at outlet means:
Less throughput
More mixing

DYE MARKER MOVEMENT EXPERIMENTS

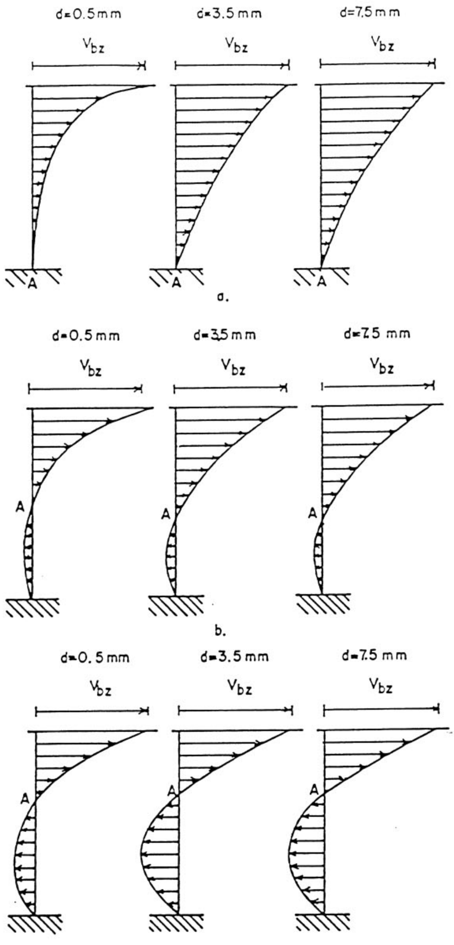
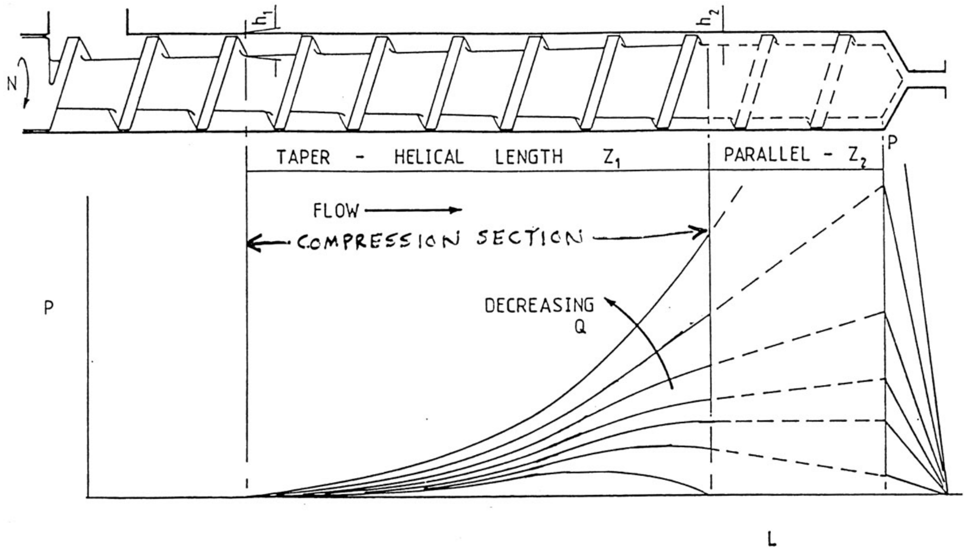


Figure 6.9 Down channel velocity profile measured by Eccher and Valentinotti[12] in the screw channel, measured at 0.5 mm, 3.5 mm, and 7.5 mm from the flight. The minor changes in the velocity profile indicate the effect of the flights. (a) Pure drag flow. (b) Combined drag and pressure flow. (c) Co mbined drag and pressure flow with closed discharge. (*Courtesy of Ref. 12*)

Extrusion
PRESSURE DISTRIBUTION



Pressure builds rapidly in compression section.

The taper in the compression section promotes mixing.

Compression Ratio $\quad h_1/h_2 \quad 2 \leq \dfrac{h_1}{h_2} \leq 4$

$h_1/h_2 = 4$ used for low throughput **compounding**
Example: PP and talc

$h_1/h_2 = 2$ used for high throughput **pumping**
Example: LDPE film blowing