Injection Molding

Figure 1: Principles of injection molding.

Injection molding cycle:

<table>
<thead>
<tr>
<th><strong>Extruder</strong></th>
<th><strong>Mold</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>Inject</td>
</tr>
<tr>
<td></td>
<td>Pack</td>
</tr>
<tr>
<td>Extrude</td>
<td>Solidify</td>
</tr>
<tr>
<td></td>
<td>gate solidifies</td>
</tr>
<tr>
<td></td>
<td>part solidifies</td>
</tr>
<tr>
<td>Open Mold</td>
<td></td>
</tr>
<tr>
<td>Eject Part</td>
<td></td>
</tr>
<tr>
<td>Close Mold</td>
<td></td>
</tr>
</tbody>
</table>
Injection Molding

Clamp unit

Injection unit

Tie bar
Nozzle
Heater bands
Water cooling channels
Hydraulic motor

Hydraulic fluid pipes
Mould
Moving half
Fixed half
Back-flow stop valve
Screw travel limit switches (adjustable)
Hydraulic fluid pipes

180 Ton Machine
Injection Molding

ECONOMICS

Injection molding machine is expensive.

Mold itself is expensive - Need **mass production** to justify these costs.

\[ N = \text{total number of parts} \]

\[ n = \text{number of parts molded in one shot} \]

\[ t = \text{cycle time} \]

Production Cost ($/part) = Material Cost + Mold Cost/N + Molding Machine Cost ($/hr) * t/n
Figure 2: The molded part cannot have any enclosed curves or the part will not eject from the mold!
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TYPES OF GATES
Injection Molding
THE INJECTION MOLDING WINDOW

Temperature

Injection Pressure

Thermal degradation

Flash

Melting

Short shot
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CENTER-GATED DISK

Figure 3: Mold used in conjunction with a constant volumetric flow rate $Q$.

Figure 4: Position of an advancing flow front.

Part 1 - Flow in the Runner

$$Q = \pi R_0^2 \frac{dz(t)}{dt}$$

$$z(t) = \frac{Q}{\pi R_0^2} t$$

Pressure builds during filling of the runner, given by the Hagen-Poiseuille Law:

$$P_i(t) = \frac{8 \mu Q}{\pi R_0^4} z(t) = \frac{8 \mu Q^2}{\pi^2 R_0^6} t$$
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At \( t = t_0 \), the runner is filled (\( z = L_r \))

\[
t_0 = \frac{\pi R_0^2 L_r}{Q} \quad P_i(t_0) = \frac{8\mu Q L_r}{\pi R_0^4}
\]

For \( t > t_0 \), the runner is full and the pressure drop along the runner is always constant:

\[
\Delta P_r = P_i - P_0 = \frac{8\mu Q L_r}{\pi R_0^4}
\]

Part 2 - Flow in the disk cavity

Figure 5: Position of the advancing front in the disk indicated by \( R^*(t) \).

\[
Q = 2H \ast 2\pi R^* \frac{dR^*}{dt} \quad R^*(t_0) = R_0
\]

\[
R^*^2 - R_0^2 = \frac{Q}{2\pi H}(t - t_0)
\]

Filling time \( t^* - t_0 = \frac{V}{Q} = \frac{2\pi H}{Q}(R^*^2 - R_0^2) \)

\[
v_z = v_\theta = 0
\]
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Continuity: \( \frac{1}{r} \frac{d}{dr} (rv_r) = 0 \)

\( \therefore v_r = \frac{C(z, t)}{r} \)

N-S: \( \frac{dP}{dr} = \mu \frac{d^2 v_r}{dz^2} = \frac{\mu d^2 C}{r \, dz^2} \)

\( \frac{dP}{dz} = \frac{dP}{d\theta} = 0 \)

\( \frac{r \, dP}{\mu \, dr} = \frac{d^2 C}{dz^2} = A(t) \)

at \( z = \pm H \quad v_r = 0 \)

at \( r = R_0 \quad P = P_0(t) \)

\[ v_r(r, z, t) = -\frac{A(t)H^2}{2r} \left[ 1 - \left( \frac{z}{H} \right)^2 \right] \]

Volumetric Flow Rate \( Q = 4\pi \int_0^H rv_r \, dz \)

\[ A = -\frac{3Q}{4\pi H^3} \quad \text{constant} \quad \therefore \text{constant} \ A \]

\[ r \frac{dP}{dr} = A\mu = -\frac{3\mu Q}{4\pi H^3} \]

\[ P_0 - P = \frac{3\mu Q}{4\pi H^3} \ln(r/R_0) \]
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The pressure drop is logarithmic

\[ P \sim \ln(1/r) \]

B.C. at \( r = R^* \quad P = 0 \)

\[ P_0 = \frac{3\mu Q}{4\pi H^3} \ln(R^*/R_0) \]

can plug in previous result for \( R^* \) to get \( P_0(t) \)
Combine with pressure drop in runner to find

\[ P_i = P_0 + \Delta P_r = \frac{3\mu Q}{4\pi H^3} \ln \left( \frac{R^*(t)}{R_0} \right) + \frac{8\mu Q L_r}{\pi R_0^4} \quad \text{for } t_0 < t < t^* \]

Figure 6: \( P^* \) is the pressure required to fill the mold.
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PACKING STAGE

When the mold is full, flow stops, so there is no longer a pressure drop.

Pressure $P^*$ is used to **pack** the mold.

Packing pressure must be maintained until the gate solidifies.

Clamping force to hold mold closed:

$$ F = \int_A P^* dA = 2\pi P^* \int_0^R r dr = \pi R^2 P^* $$

General Result $F = P^* A$

Example: Typical packing pressure $P^* = 10^8$ Pa for a total area of $A = 0.1$ m$^2$. Clamp force $F = P^* A = 10^7$ Nt= 1000 tons.

This is why injection molding machines are so large. They have to keep the mold closed!
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SIZING AN INJECTION MOLDING MACHINE

Packing pressure $\approx 10^8$ Pa

Clamping force $F = P \times A$

![Graph showing clamping force as a function of surface area.](image)

Figure 7: Clamping force as a function of surface area. Note: logarithmic scales.

Mold a single tensile bar - 50 ton machine

Mold a front end of a car - 5000 ton machine

“Typical” sizes are 100-1000 tons

For complicated parts $A = \text{projected area}$
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CRITIQUE OF OUR MOLD-FILLING CALCULATION

Our calculation was fairly nasty, yet we made so many assumptions that the calculation is useless quantitatively.

Assumptions:

1. Constant volumetric flow rate - otherwise keep time derivatives in the three Navier-Stokes Equations.

2. Negligible pressure drop in gate

3. Newtonian - Polymer melts are not Newtonian! This assumption keeps the three Navier-Stokes Equations linear.

4. Isothermal - This is the worst assumption. Actually inject hot polymer into a cold mold to improve cycle time. To include heat transfer, another coupled PDE is needed! The coupling is non-trivial because during injection, a skin of cold polymer forms on the walls of the mold and grows thicker with time.