## Injection Molding



Figure 1: Principles of injection molding.

Injection molding cycle:

| Extruder | Mold |  |
| :---: | :---: | :---: |
| Pressure | Inject |  |
|  | Pack | gate solidifies |
| Extrude | Solidify |  |
|  |  | part solidifies |
| Open Mold |  |  |
|  | Eject Part |  |
|  | Close Mold |  |



## Injection Molding ECONOMICS

Injection molding machine is expensive.
Mold itself is expensive - Need mass production to justify these costs.

$$
N=\text { total number of parts }
$$

$n=$ number of parts molded in one shot

$$
t=\text { cycle time }
$$

Production Cost (\$/part) $=$ Material Cost
+Mold Cost/N

+ Molding Machine Cost $(\$ / \mathrm{hr}) * t / n$


## Injection Molding EJECTION



Figure 2: The molded part cannot have any enclosed curves or the part will not eject from the mold!

## Injection Molding TYPES OF GATES



TAB GATE


FILM TYPE GATE

SPOKE, SPIDER CR LEG GATE


PIN POINT TAB GATE


SUBMARINE FLARE GATE CHISEL GATE

# Injection Molding <br> THE INJECTION MOLDING WINDOW 



## Injection Molding CENTER-GATED DISK



Figure 3: Mold used in conjunction with a constant volumetric flow rate $Q$.


Figure 4: Position of an advancing flow front.

Part 1 - Flow in the Runner

$$
\begin{gathered}
Q=\pi R_{0}^{2} \frac{d z(t)}{d t} \\
z(t)=\frac{Q}{\pi R_{0}^{2}} t
\end{gathered}
$$

Pressure builds during filling of the runner, given by the Hagen-Poiseuille Law:

$$
P_{i}(t)=\frac{8 \mu Q}{\pi R_{0}^{4}} z(t)=\frac{8 \mu Q^{2}}{\pi^{2} R_{0}^{6}} t
$$

## Injection Molding CENTER-GATED DISK

At $t=t_{0}$, the runner is filled $\left(z=L_{r}\right)$

$$
t_{0}=\frac{\pi R_{0}^{2} L_{r}}{Q} \quad P_{i}\left(t_{0}\right)=\frac{8 \mu Q L_{r}}{\pi R_{0}^{4}}
$$

For $t>t_{0}$, the runner is full and the pressure drop along the runner is always constant:

$$
\Delta P_{r}=P_{i}-P_{0}=\frac{8 \mu Q L_{r}}{\pi R_{0}^{4}}
$$

Part 2 - Flow in the disk cavity


Figure 5: Position of the advancing front in the disk indicated by $R^{*}(t)$.

$$
\begin{gathered}
Q=2 H * 2 \pi R^{*} \frac{d R^{*}}{d t} \quad R^{*}\left(t_{0}\right)=R_{0} \\
R^{* 2}-R_{0}^{2}=\frac{Q}{2 \pi H}\left(t-t_{0}\right)
\end{gathered}
$$

Filling time $\quad t^{*}-t_{0}=\frac{V}{Q}=\frac{2 \pi H}{Q}\left(R^{* 2}-R_{0}^{2}\right)$

$$
v_{z}=v_{\theta}=0
$$

## Injection Molding CENTER-GATED DISK

$$
\begin{gathered}
\text { Continuity: } \frac{1}{r} \frac{d}{d r}\left(r v_{r}\right)=0 \\
\therefore v_{r}=\frac{C(z, t)}{r} \\
\text { N-S: } \frac{d P}{d r}=\mu \frac{d^{2} v_{r}}{d z^{2}}=\frac{\mu}{r} \frac{d^{2} C}{d z^{2}} \\
\frac{d P}{d z}=\frac{d P}{d \theta}=0 \\
\frac{r}{\mu} \frac{d P}{d r}=\frac{d^{2} C}{d z^{2}}=A(t) \\
\text { at } z= \pm H \quad v_{r}=0 \\
\text { at } r=R_{0} \quad P=P_{0}(t) \\
v_{r}(r, z, t)=-\frac{A(t) H^{2}}{2 r}\left[1-\left(\frac{z}{H}\right)^{2}\right] \\
\text { Volumetric Flow Rate } Q=4 \pi \int_{0}^{H} r v_{r} d z
\end{gathered}
$$

$$
A=-\frac{3 Q}{4 \pi H^{3}} \quad \text { constant } Q \quad \therefore \text { constant } A
$$

$$
\begin{gathered}
r \frac{d P}{d r}=A \mu=-\frac{3 \mu Q}{4 \pi H^{3}} \\
P_{0}-P=\frac{3 \mu Q}{4 \pi H^{3}} \ln \left(r / R_{0}\right)
\end{gathered}
$$

## Injection Molding CENTER-GATED DISK

The pressure drop is logarithmic

$$
P \sim \ln (1 / r)
$$

B.C. at $r=R^{*} \quad P=0$

$$
P_{0}=\frac{3 \mu Q}{4 \pi H^{3}} \ln \left(R^{*} / R_{0}\right)
$$

can plug in previous result for $R^{*}$ to get $P_{0}(t)$
Combine with pressure drop in runner to find

$$
P_{i}=P_{0}+\Delta P_{r}=\frac{3 \mu Q}{4 \pi H^{3}} \ln \left(\frac{R^{*}(t)}{R_{0}}\right)+\frac{8 \mu Q L_{r}}{\pi R_{0}^{4}} \quad \text { for } t_{0}<t<t^{*}
$$



Figure 6: $P^{*}$ is the pressure required to fill the mold.

## Injection Molding PACKING STAGE

When the mold is full, flow stops, so there is no longer a pressure drop.
Pressure $P^{*}$ is used to pack the mold.
Packing pressure must be maintained until the gate solidifies.
Clamping force to hold mold closed:

$$
F=\int_{A} P^{*} d A=2 \pi P^{*} \int_{0}^{R} r d r=\pi R^{2} P^{*}
$$

General Result $F=P^{*} A$
Example: Typical packing pressure $P^{*}=10^{8} \mathrm{~Pa}$ for a total area of $A=0.1 \mathrm{~m}^{2}$. Clamp force $F=P^{*} A=10^{7} \mathrm{Nt}=1000$ tons.

This is why injection molding machines are so large. They have to keep the mold closed!

## Injection Molding SIZING AN INJECTION MOLDING

Packing pressure $\cong 10^{8} \mathrm{~Pa}$
Clamping force $F=P^{*} A$


Figure 7: Clamping force as a function of surface area. Note: logarithmic scales.

Mold a single tensile bar - 50 ton machine
Mold a front end of a car - 5000 ton machine
"Typical" sizes are 100-1000 tons
For complicated parts $A=$ projected area

# Injection Molding CRITIQUE OF OUR MOLD-FILLING CALCULATION 

Our calculation was fairly nasty, yet we made so many assumptions that the calculation is useless quantitatively.

## Assumptions:

1. Constant volumetric flow rate - otherwise keep time derivatives in the three Navier-Stokes Equations.
2. Negligible pressure drop in gate
3. Newtonian - Polymer melts are not Newtonian! This assumption keeps the three Navier-Stokes Equations linear.
4. Isothermal - This is the worst assumption. Actually inject hot polymer into a cold mold to improve cycle time. To include heat transfer, another coupled PDE is needed! The coupling is non-trivial because during injection, a skin of cold polymer forms on the walls of the mold and grows thicker with time.
