Injection Molding

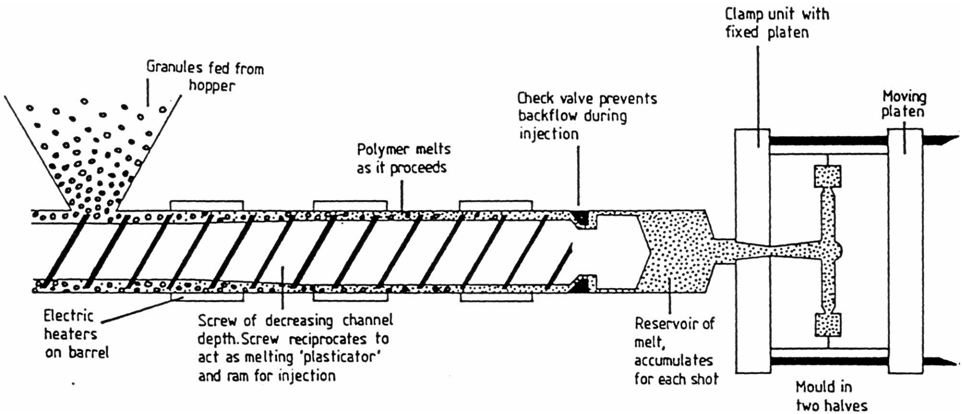
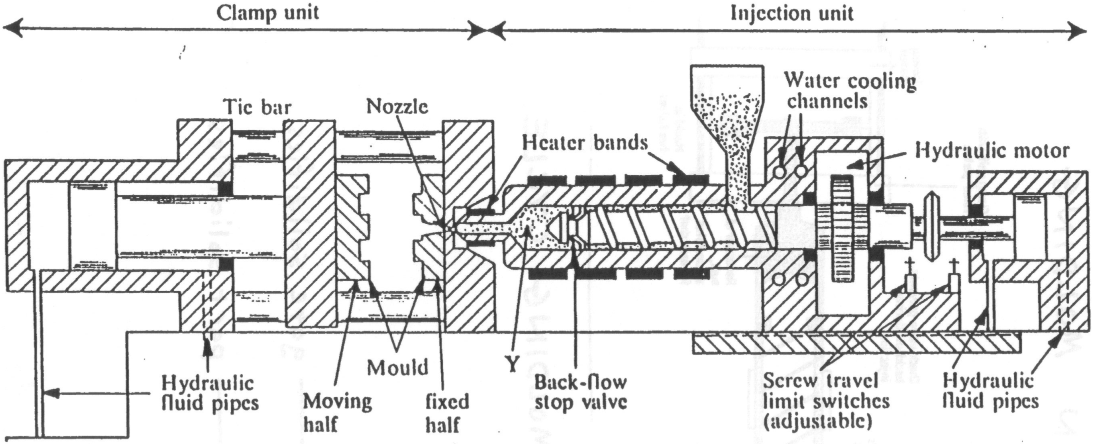


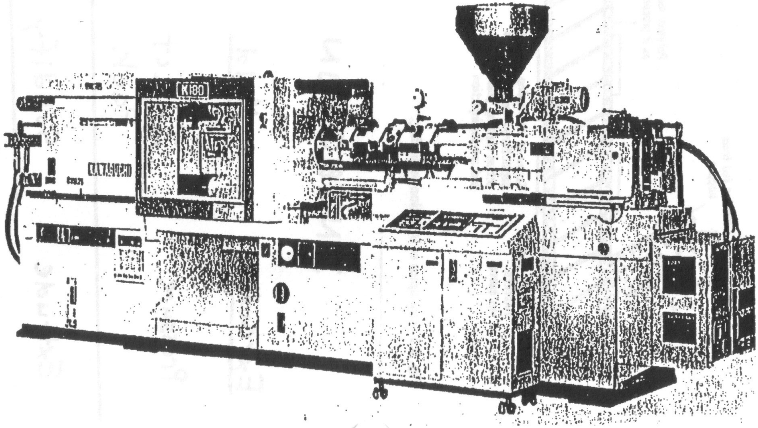
Figure 1: Principles of injection molding.

Injection molding cycle:

Extruder	***Mold***	
Pressure	Inject	
	Pack	gate solidifies
Extrude	Solidify	
		part solidifies
	Open Mold	
	Eject Part	
	Close Mold	

Injection Molding





180 Ton Machine

Injection Molding ECONOMICS

Injection molding machine is expensive.

Mold itself is expensive - Need **mass production** to justify these costs.

$$N = \text{total number of parts}$$

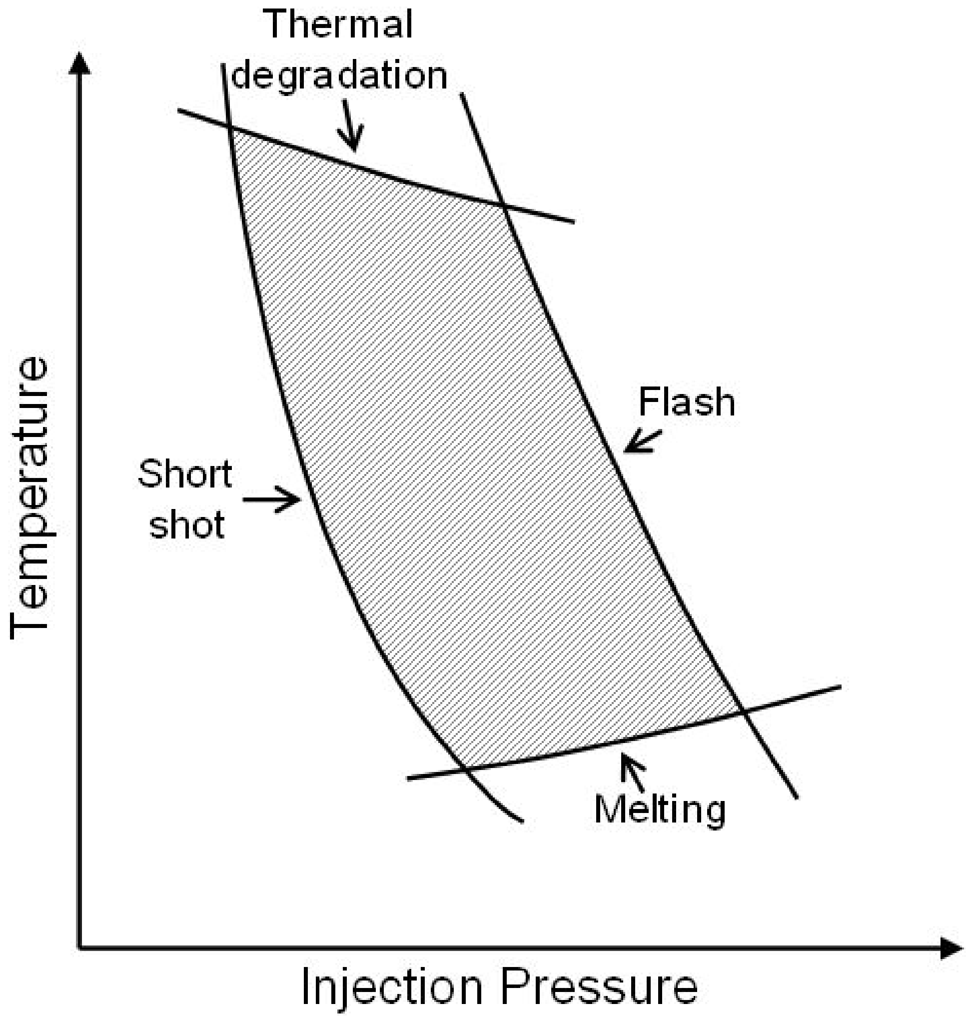
$$n = \text{number of parts molded in one shot}$$

$$t = \text{cycle time}$$

$$\begin{aligned} \text{Production Cost (\$/part)} \quad = \quad & \text{Material Cost} \\ & + \text{Mold Cost}/N \\ & + \text{Molding Machine Cost (\$/hr)} * t/n \end{aligned}$$

Injection Molding

THE INJECTION MOLDING WINDOW



Injection Molding
PACKING STAGE

When the mold is full, flow stops, so there is no longer a pressure drop.

Pressure P^* is used to **pack** the mold.

Packing pressure must be maintained until the gate solidifies.

Clamping force to hold mold closed:

$$F = \int_A P^* dA = 2\pi P^* \int_0^R r dr = \pi R^2 P^*$$

General Result $F = P^* A$

Example: Typical packing pressure $P^* = 10^8$ Pa for a total area of $A = 0.1$ m^2. Clamp force $F = P^* A = 10^7$ Nt= 1000 tons.

This is why injection molding machines are so large. They have to keep the mold closed!

Injection Molding
SIZING AN INJECTION MOLDING MACHINE

Packing pressure $\cong 10^8$Pa

Clamping force $F = P^* A$

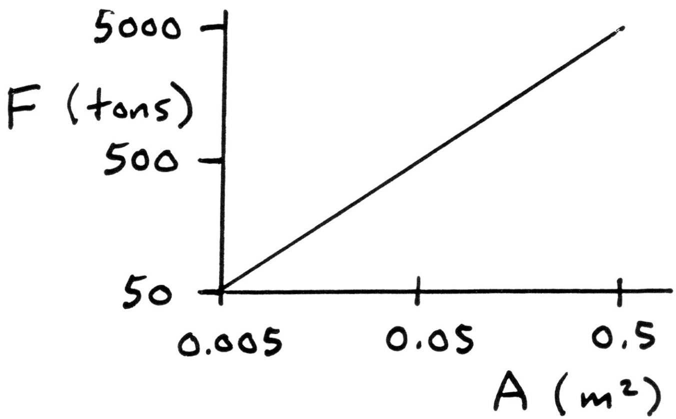


Figure 7: Clamping force as a function of surface area. Note: logarithmic scales.

Mold a single tensile bar - 50 ton machine

Mold a front end of a car - 5000 ton machine

"Typical" sizes are 100-1000 tons

For complicated parts A = projected area

Injection Molding
CRITIQUE OF OUR MOLD-FILLING CALCULATION

Our calculation was fairly nasty, yet we made so many assumptions that the calculation is *useless* quantitatively.

Assumptions:

1. Constant volumetric flow rate - otherwise keep time derivatives in the three Navier-Stokes Equations.

2. Negligible pressure drop in gate

3. Newtonian - Polymer melts are **not** Newtonian! This assumption keeps the three Navier-Stokes Equations linear.

4. **Isothermal** - This is the worst assumption. Actually inject hot polymer into a cold mold to improve cycle time. To include heat transfer, another coupled PDE is needed! The coupling is non-trivial because during injection, a skin of cold polymer forms on the walls of the mold and grows thicker with time.

Injection Molding
BALANCING RUNNER SYSTEMS

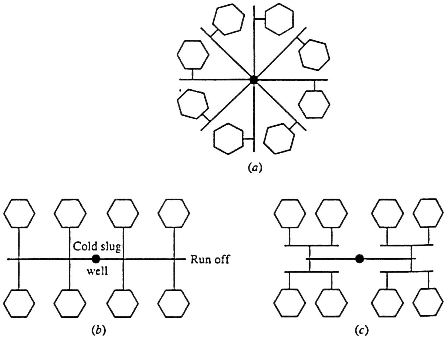


Figure 1: Two naturally balanced (symmetric) runner systems and one counter-example.

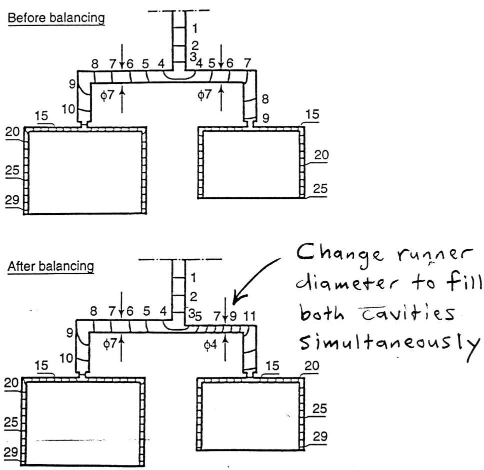


Figure 2: An artificially balanced runner system.

Injection Molding
CONSEQUENCE OF IMBALANCED RUNNER SYSTEMS

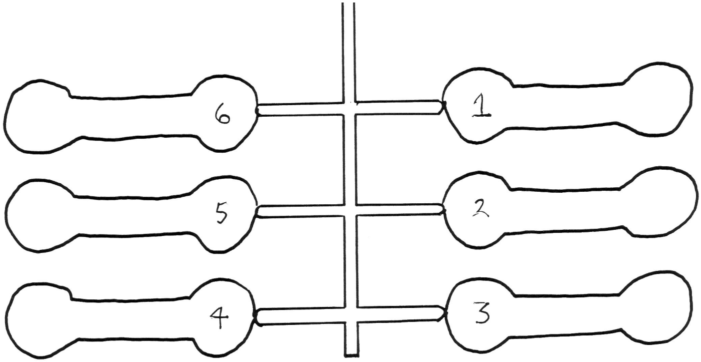


Figure 3: Need to **overpack** 1 and 6 to fill 3 and 4.

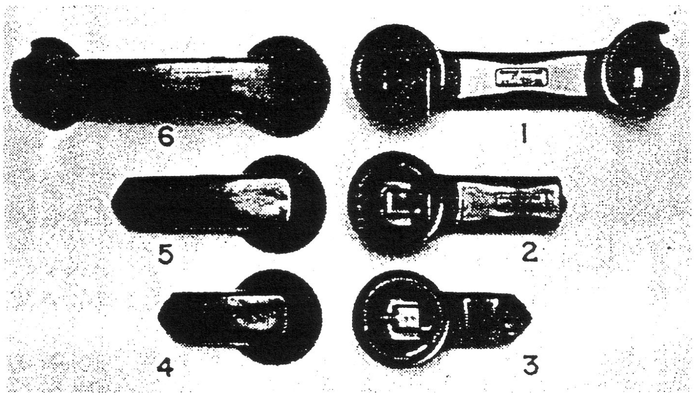


Figure 4: Short shots in a telephone-handle molding die.

Injection Molding
INCOMPRESSIBLE CONTINUITY EQUATION FOR LIQUIDS

Cartesian coordinates: x, y, z

$$\frac{dV_x}{dx} + \frac{dV_y}{dy} + \frac{dV_z}{dz} = 0$$

Cylindrical coordinates: r, θ, z

$$\frac{1}{r}\frac{d}{dr}(rv_r) + \frac{1}{r}\frac{dv_\theta}{d\theta} + \frac{dv_z}{dz} = 0$$

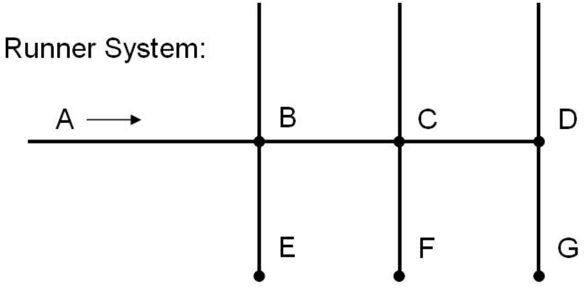
Spherical coordinates: r, θ, ϕ

$$\frac{1}{r^2}\frac{d}{dr}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{d}{d\theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{dv_\phi}{d\phi} = 0$$

All are simply $\vec{\nabla} \cdot \vec{v} = 0$

Injection Molding

Example: use Hagen-Poiseuille Law to balance the runners



$$\text{Hagen-Poiseuille Law: } \Delta P = \frac{8\mu L Q}{\pi R^4}$$

Suppose: $R_{AB} = R_{BC} = R_{CD} = R_{DG} \equiv R$

What size do we make R_{BE} and R_{CF} to balance the pressures at E, F and G?

$$\text{Flow is split 6 ways:} \qquad Q_{AB} \equiv Q$$

$$Q_{BC} = \frac{2}{3} Q$$

$$Q_{CD} = \frac{1}{3} Q$$

$$Q_{BE} = Q_{CF} = Q_{DG} = \frac{1}{6} Q$$

All lengths are equal, define $K \equiv 8\mu L / \pi$

Injection Molding

Pressure drops are additive:

$$\begin{aligned}\Delta P_{BG} &= \frac{KQ_{BC}}{R_{BC}^4} + \frac{KQ_{CD}}{R_{CD}^4} + \frac{KQ_{DG}}{R_{DG}^4} \\ &= \frac{2KQ}{3R^4} + \frac{KQ}{3R^4} + \frac{KQ}{6R^4} \\ &= \frac{7KQ}{6R^4}\end{aligned}$$

$$\begin{aligned}\Delta P_{BF} &= \frac{KQ_{BC}}{R_{BC}^4} + \frac{KQ_{CF}}{R_{CF}^4} \\ &= \frac{2}{3}\frac{KQ}{R^4} + \frac{KQ}{6R_{CF}^4}\end{aligned}$$

$$\text{First Result: } \Delta P_{BG} = \Delta P_{BF} \quad \Rightarrow \quad \frac{1}{6R_{CF}^4} = \frac{1}{2R^4}$$

$$R_{CF} = \frac{R}{3^{1/4}} = 0.76R$$

$$\Delta P_{BE} = \frac{KQ}{6R_{BE}^4}$$

$$\text{Second Result: } \Delta P_{BE} = \Delta P_{BG} \quad \Rightarrow \quad \frac{1}{6R_{BE}^4} = \frac{7}{6R^4}$$

$$R_{BE} = \frac{R}{7^{1/4}} = 0.61R$$

Injection Molding
EXTREME EXAMPLE OF RUNNER BALANCING

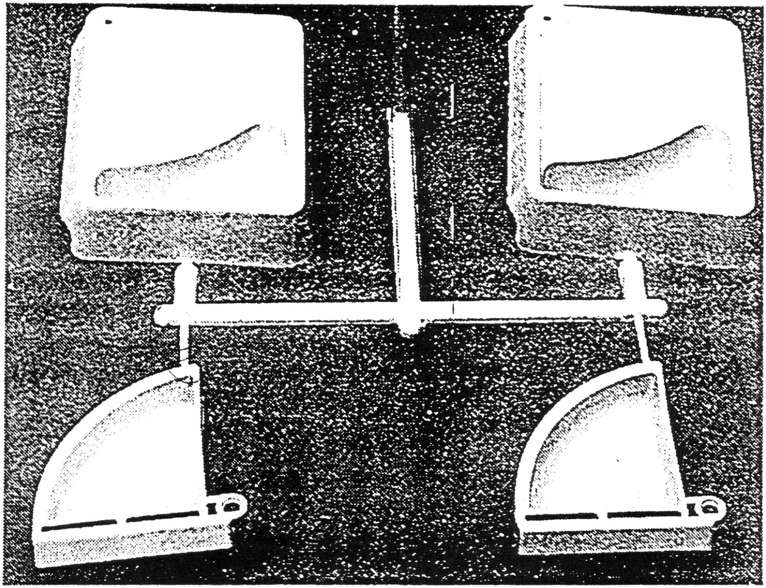
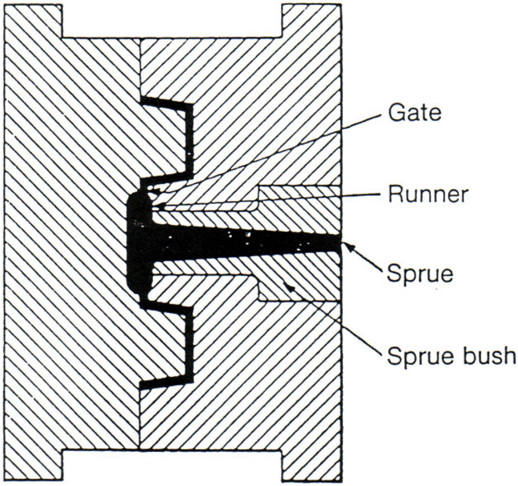


Figure 5: Family mold (pair of dishwasher detergent holding set).

Injection Molding
CONVENTIONAL INJECTION MOLDING



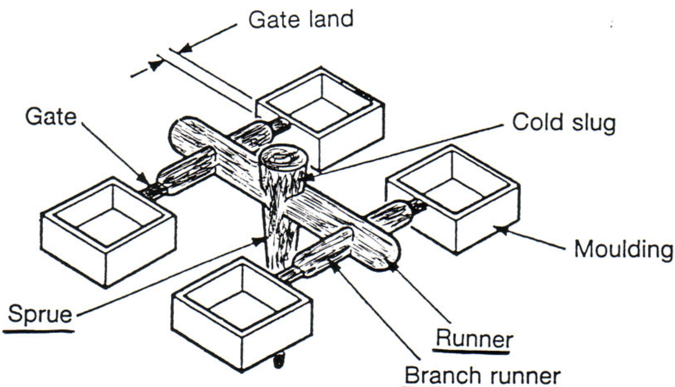


Figure 6: Discard or regrind.

Injection Molding
INJECTION MOLDING DEFECTS

Weld lines

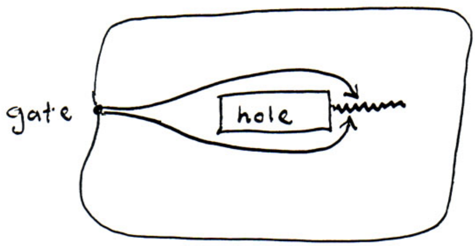


Figure 7: Cold flow fronts recombine to make a visible line that can be mechanically weak.

Voids, **Sink Marks**, **Shrinkage**

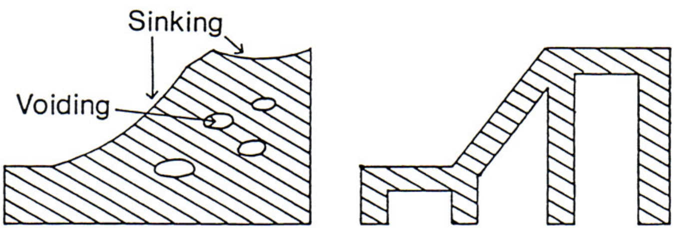


Figure 8: Use of ribs instead of a solid section. Solid section (left) and thin section (right). 10% shrink can be expected.

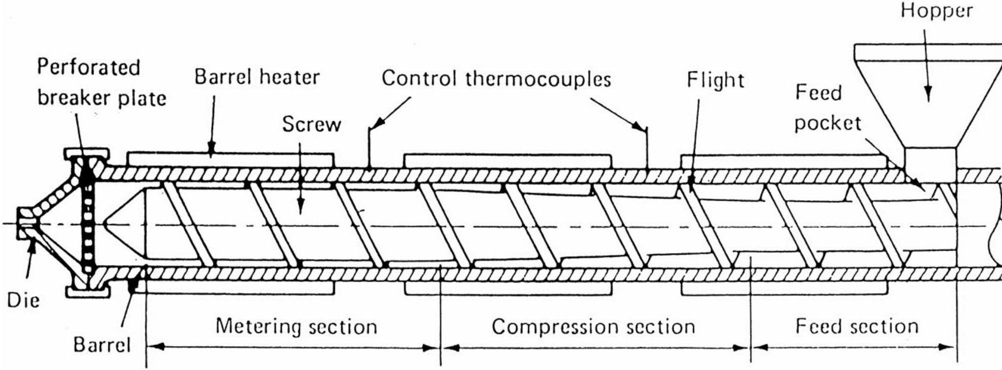
Thick sections cool after gate freezes.

Sticking - Injection pressure too high (overpack).

Warping - Insufficient cooling before ejection.

Burning - Extrusion temperature too high. Shear heating.

Extrusion



Extruders are sized by their barrel diameter D.

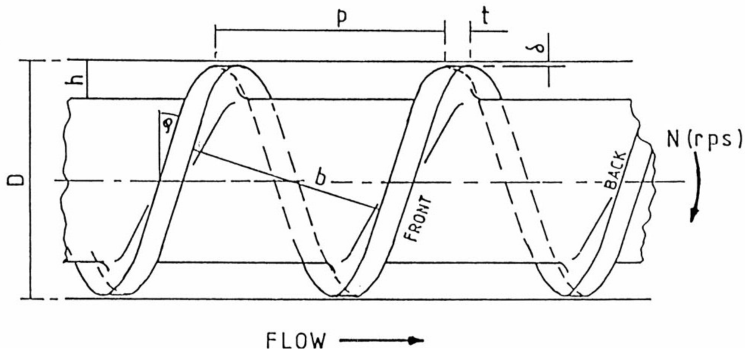


Figure 1: An extruder has two roles: Pumping & Mixing.

Extrusion

Unwind helical screw into a flat cartesian coordinate system.

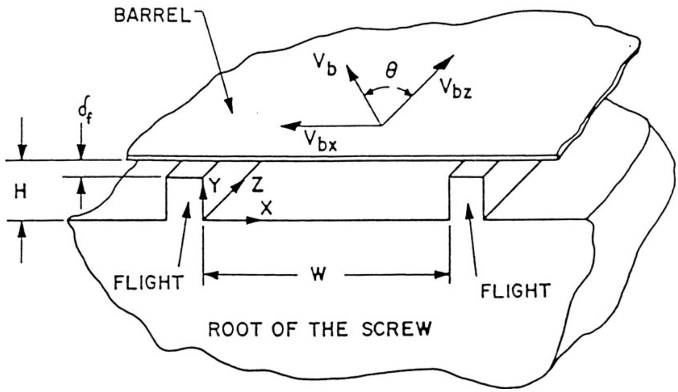


Figure 2: The extruder has the screw turning in a fixed barrel.

Choose coordinate system that moves with the screw. Then effectively have the barrel moving with velocity $\vec{v}_b$.

$$\vec{v}_b = -v_b \sin\theta \vec{i} + v_b \cos\theta \vec{k} \qquad v_b = |\vec{v}_b|$$

Time independent

$$v_y = 0 \qquad v_x = v_x(x,y) \qquad v_z = v_z(x,y)$$

Continuity

$$\frac{dv_x}{dx} = 0 \qquad \Rightarrow \qquad v_x = v_x(y)$$

N.-S.:

$$\frac{dP}{dx} = \mu \frac{d^2 v_x}{dy^2} \qquad \frac{dP}{dy} = 0$$

$$\frac{dP}{dz} = \mu \left(\frac{d^2 v_z}{dx^2} + \frac{d^2 v_z}{dy^2} \right)$$

B.C. at $y = 0$, $\quad \vec{v} = 0$, $\quad v_x = v_z = 0$
at $y = H$, $\vec{v} = \vec{v}_b$, $v_x = -v_b \sin\theta$, $v_z = v_b \cos\theta$

Extrusion

$$v_x = v_b \sin\theta \frac{y}{H}\left[2 - 3\frac{y}{h}\right]$$

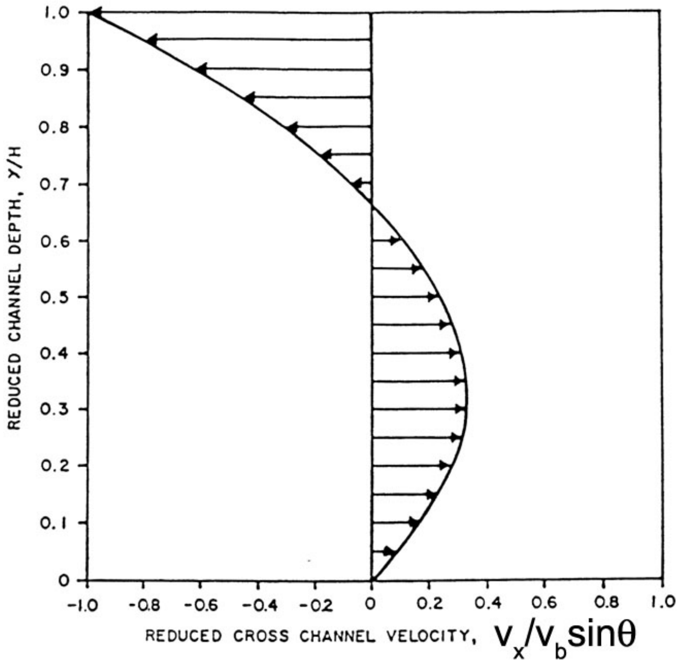


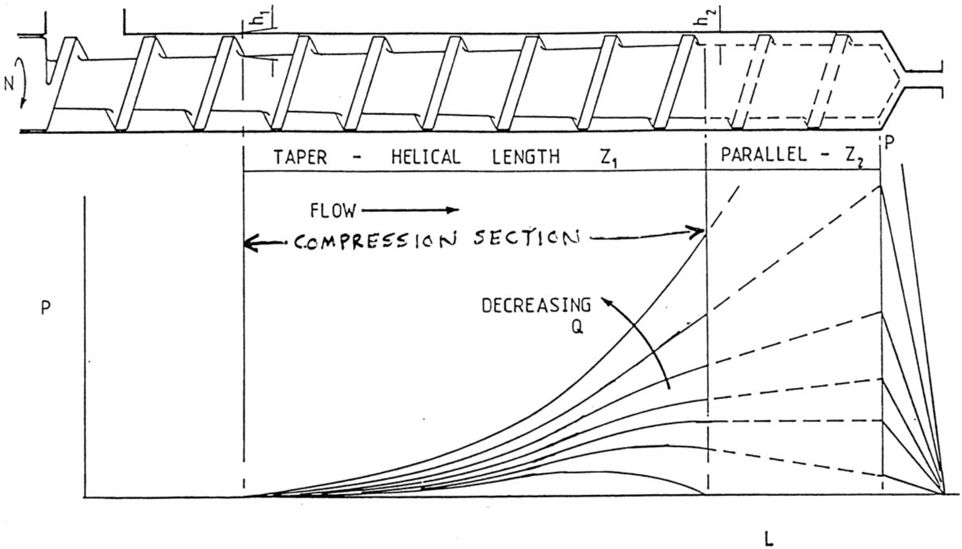
Figure 3: The extruder has the screw turning in a fixed barrel.

v_x is a **universal function** of $v_b \sin\theta$ and y/H.

independent of viscosity!

at $\quad y = \frac{2}{3}H, \qquad v_x = 0 \qquad$ Observed in experiment

Extrusion
PRESSURE DISTRIBUTION



Pressure builds rapidly in compression section.

The taper in the compression section promotes mixing.

$$\text{Compression Ratio} \qquad h_1/h_2 \qquad 2 \leq \frac{h_1}{h_2} \leq 4$$

$h_1/h_2 = 4$ used for low throughput **compounding**
Example: PP and talc

$h_1/h_2 = 2$ used for high throughput **pumping**
Example: LDPE film blowing

Single-Screw Extrusion
THE EXTRUDER CHARACTERISTIC

A. W. Birley, B. Haworth and J. Batchelor, Physics of Plastics: Processing, Properties and Materials Engineering, Hanser (1992) Chapter 4.
(on reserve in Deike Library)

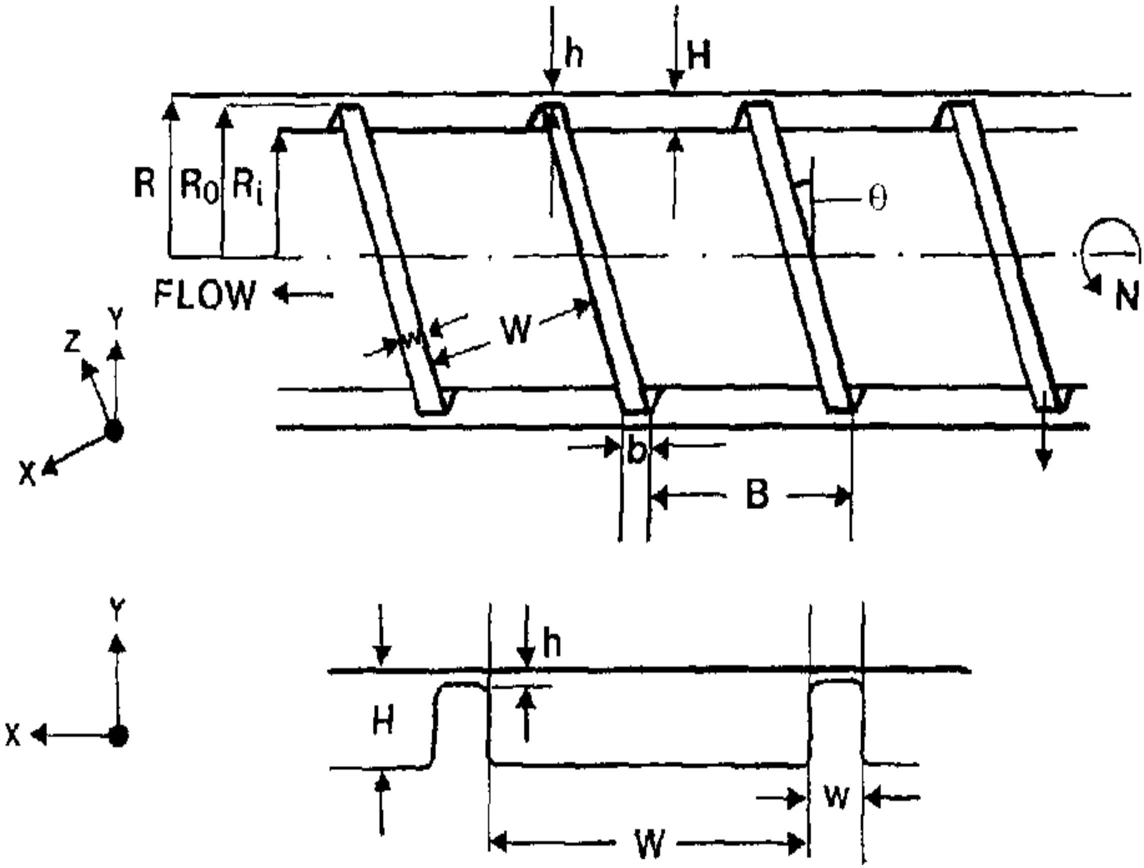


Figure 1: Definitions of Symbols

Barrel Diameter $D = 2R$	Channel Depth $H = R - R_i$
Screw Helix Angle θ	Screw Clearance $h = R - R_o$
Screw Pitch $B + b$	Channel Width W
Screw Rotation Speed N (RPM)	Flight Width w

Single-Screw Extrusion
THE EXTRUDER CHARACTERISTIC

DRAG FLOW — the Couette flow between the rotating screw and the stationary barrel

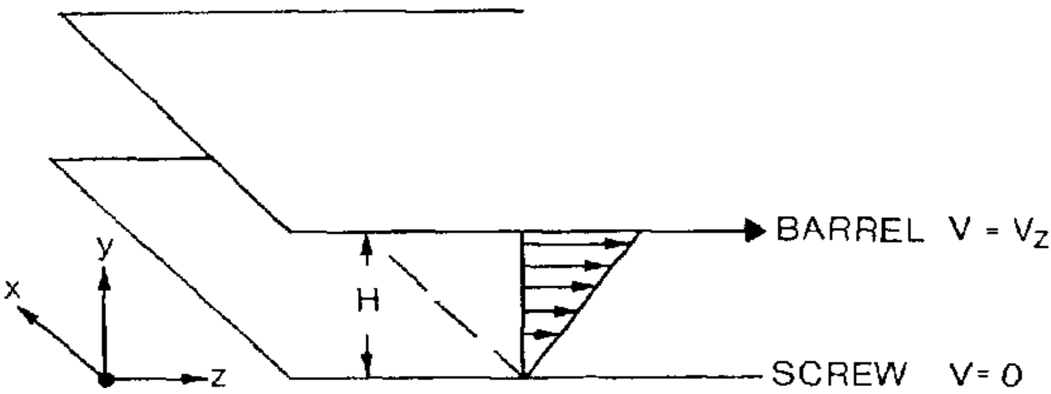


Figure 2: Drag Flow Mechanism

Down Channel Velocity Component $$V_z = V\cos\theta \tag{4.1}$$

Volumetric Flow Rate from Drag $$Q_D = W\int_0^H v(y)dy \tag{4.2}$$

For a Newtonian fluid, the velocity profile is linear:

$$v(y) = V_z\frac{y}{H}$$

$$Q_D = \frac{WV_z}{H}\int_0^H ydy = \frac{WV_z}{H}\frac{H^2}{2} = \frac{WV_zH}{2} \tag{4.3}$$

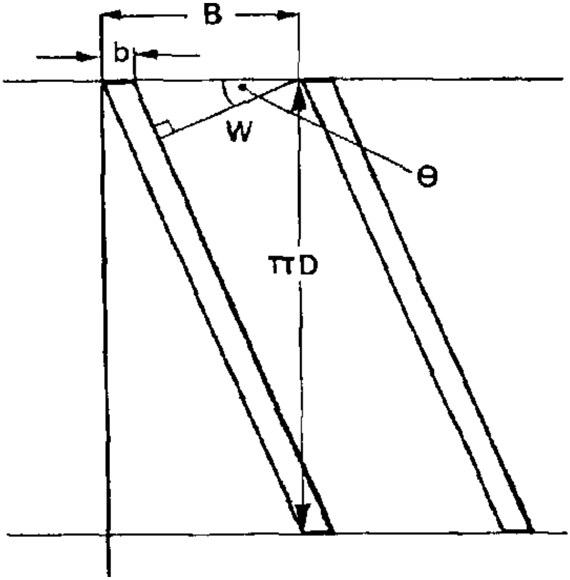


Figure 3: Unrolled Single Turn of the Extruder Screw Helix

The tangential velocity at the barrel surface is determined from the rotation speed of the screw:

$$V = \pi DN \tag{4.4}$$

Down Channel Velocity Component $\quad V_z = \pi DN\cos\theta \quad (4.5)$

$$Q_D = \frac{\pi}{2}WHDN\cos\theta \equiv \alpha N \tag{4.6}$$

The drag flow effectively pumps the polymer through the extruder.

Q_D is proportional to the rotation speed N.

Proportionality constant α only depends on screw geometry.

Single-Screw Extrusion
THE EXTRUDER CHARACTERISTIC

PRESSURE FLOW — the Poiseuille flow suppressing flow through the extruder

Extruders usually have some FLOW RESTRICTION (like a die) at the end of the extruder. This creates a pressure gradient along the screw that works against the flow through the screw:

$$Q_P = -\frac{WH^3}{12\mu}\frac{\Delta P}{L} \equiv -\frac{\beta}{\mu}\Delta P \tag{4.7}$$

Again, the proportionality constant β only depends on screw geometry.

The NET VOLUMETRIC FLOW RATE is the sum:

$$Q = Q_D + Q_P \tag{4.8}$$

Example 1: OPEN DISCHARGE
No flow restriction at the end of the extruder (remove die)

$$Q_P = 0 \qquad \text{and} \qquad Q = Q_D$$

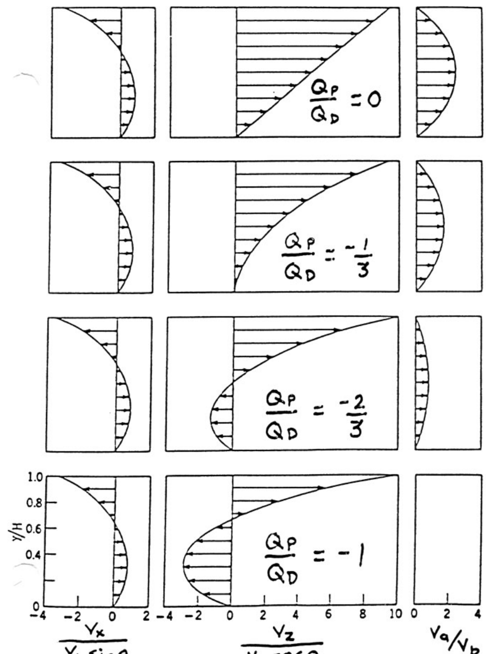
Example 2: CLOSED DISCHARGE
No flow out of the extruder (plug die)

$$Q = 0 \qquad , \qquad Q_P = Q_D \qquad \text{and} \qquad \Delta P = \alpha\mu N/\beta$$

Extrusion

Velocity along the axial direction of the extruder is a vector sum of v_x and v_z.

$$v_a = v_x \cos\theta + v_z \sin\theta$$



Cross channel, down channel, and axial velocity profiles for various Q_p/Q_c values.

No flow restriction (pure drag flow)
Optimum Pumping

$v_z = v_b \cos\theta (y/H)^2$

$v_z = 0$ at $y = 0$ and at $y = H/2$

Plugged extruder outlet (zero net flow). Optimum mixing.

Single-Screw Extrusion
THE EXTRUDER CHARACTERISTIC

In general the die restricts the flow somewhat, but not completely. Combining equations 4.6, 4.7, and 4.8, we get the EXTRUDER CHARACTERISTIC:

$$Q = \alpha N - \frac{\beta}{\mu}\Delta P \tag{4.13}$$

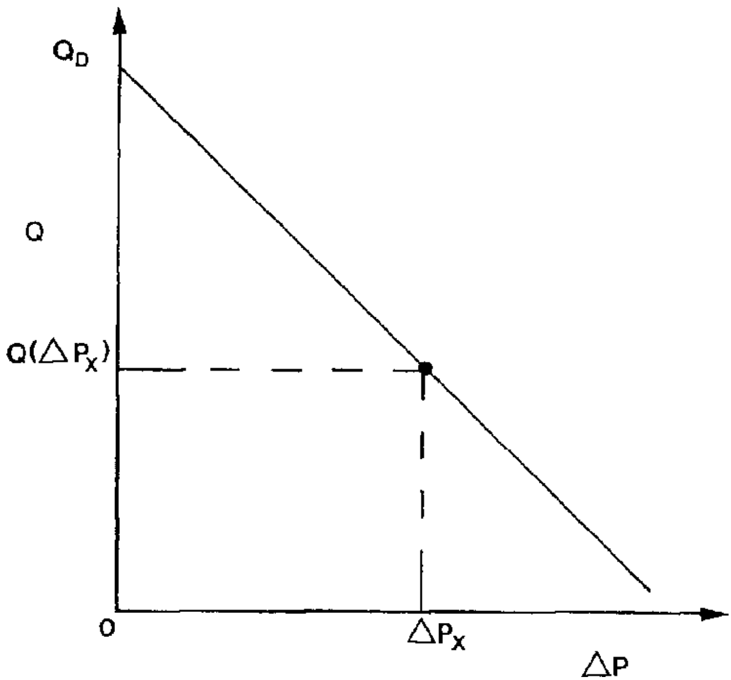


Figure 4: The Extruder Characteristic for a Newtonian Fluid is a linear relation between Q and ΔP.

y-axis intercept ⇒ OPEN DISCHARGE ($\Delta P = 0$)
x-axis intercept ⇒ CLOSED DISCHARGE ($Q = 0$)

More Flow Restriction ⇒
Larger Pressure (larger ΔP) ⇒
Smaller Throughput (lower Q)

Single-Screw Extrusion
THE DIE CHARACTERISTIC

There is a simple relation between pressure drop and volumetric flow rate in the die.

$$Q = K\frac{\Delta P}{\mu} \tag{4.21}$$

Circular Die: $K = \dfrac{\pi R^4}{8L}$ Hagen-Poiseuille Law

Slit Die: $K = \dfrac{WH^3}{12L}$

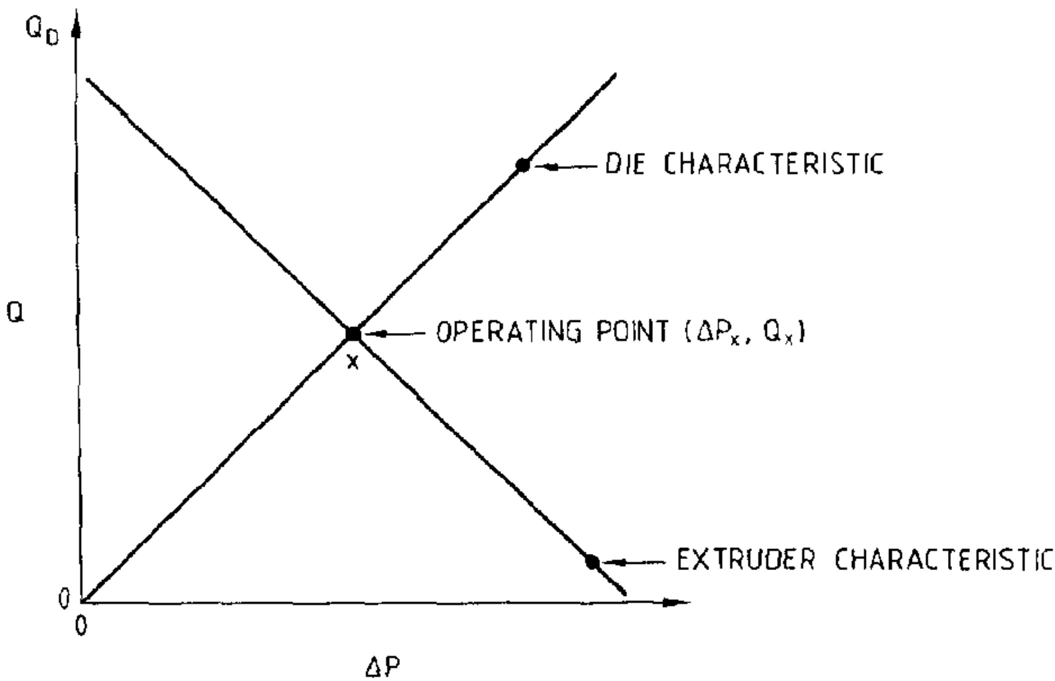


Figure 5: The Operating Point is the Intersection of the Extruder Characteristic and the Die Characteristic.

Single-Screw Extrusion
EFFECT OF PROCESS VARIABLES

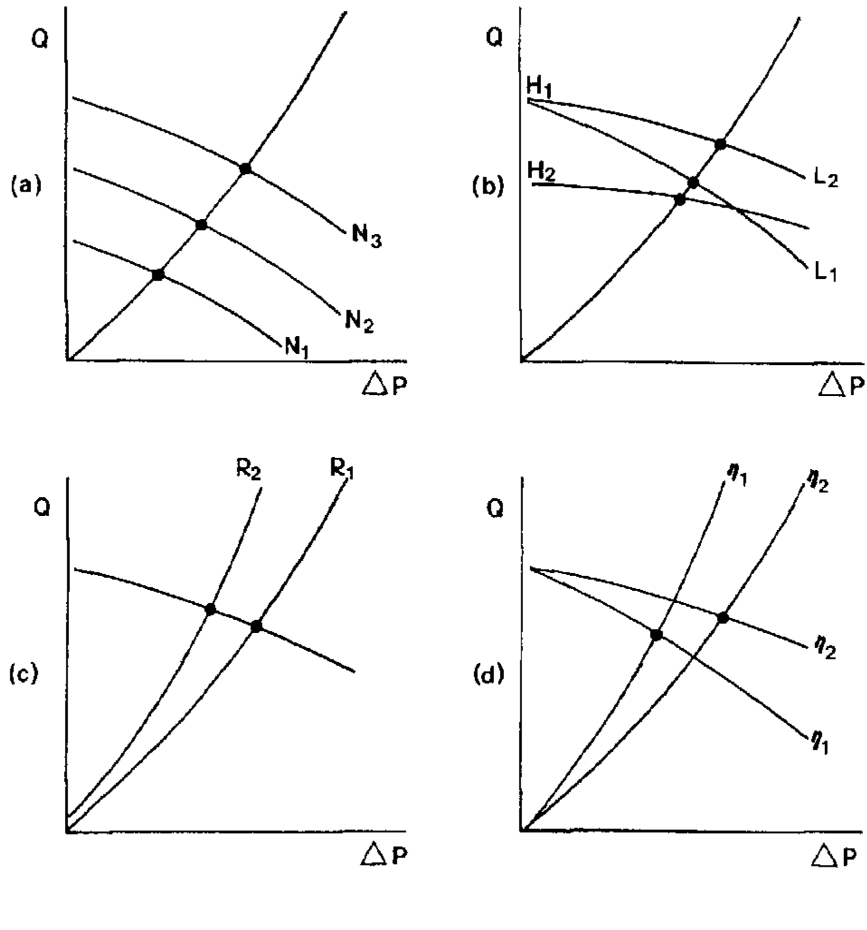
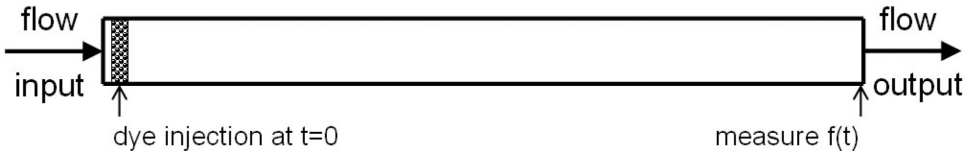


Figure 6: (a) Effect of Screw Speed ($N_3 > N_2 > N_1$).
(b) Effect of Screw Channel Depth ($H_1 > H_2$)
and Metering Section Length ($L_2 > L_1$).
(c) Effect of Die Radius ($R_2 > R_1$).
(d) Effect of Viscosity ($\eta_2 > \eta_1$).

Residence Time Distribution

Without disturbing the steady-state flow, insert a dye marker uniformly across the cross sectional area of the input, at time $t = 0$.



What is the concentration of dye exiting the flow as a function of time?

Dye Concentration at Exit $f(t)$ (amount per unit time)

Residence Time t (time dye takes to exit)

Mean Residence Time $\bar{t} = \int_0^\infty t f(t) dt$

Example: Newtonian flow in a circular pipe

$$P_1 > P_2 \qquad v_r = v_\theta = 0 \qquad v_z = \frac{\Delta P}{4\mu L}\left[R^2 - r^2\right]$$

Residence time depends on radial position because velocity depends on radial position.

$$t = \frac{L}{v_z} = \frac{4\mu L^2}{\Delta P[R^2 - r^2]}$$

Shortest residence time at centerline of pipe because the maximum velocity is there.

$$t_0 = \frac{L}{v_{max}} = \frac{4\mu L^2}{\Delta P R^2} \qquad (t \text{ at } r = 0)$$

Residence Time Distribution

Mean residence time:

$$\bar{t} = \int_0^\infty t f(t) dt = \int_{t_0}^\infty \frac{2t_0^2 dt}{t^2} = -\frac{2t_0^2}{t}\bigg|_{t_0}^\infty$$

$$\bar{t} = 2t_0$$

Comparison of residence time distributions

For pipe flow: $\bar{t} f(t) = 4(t_0/t)^3 = (\bar{t}/t)^3/2$

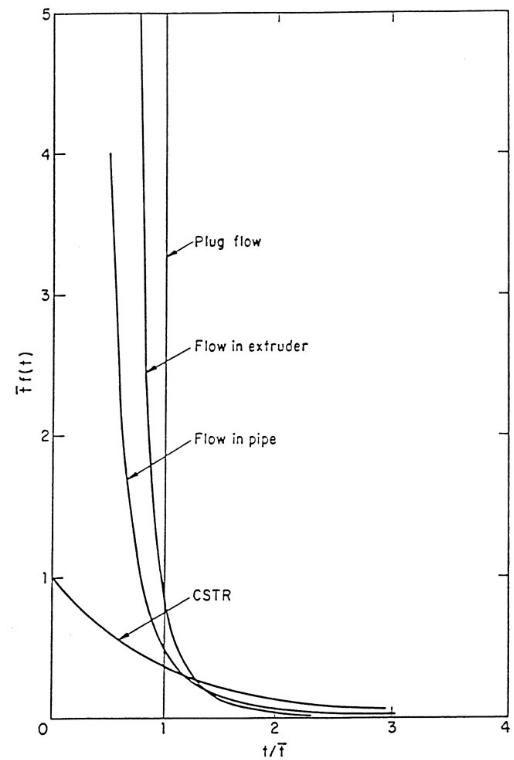


Figure 7.6 The residence time distribution function $t_f(t)$ versus reduced time $t/\bar{t}$ for flow in an extruder, compared to (a) plug flow (where all fluid particles have equal residence time), (b) isothermal flow of Newtonian fluid in cylindrical pipes, and (c) continuous stirred tank reactor. The basis of comparison was the equal mean residence time, $\bar{t}$, in all cases.

Figure 1: Extruder flow has a narrower residence time distribution than pipe flow because the extruder has cross-channel flows and thus improved mixing.

TWIN-SCREW EXTRUSION

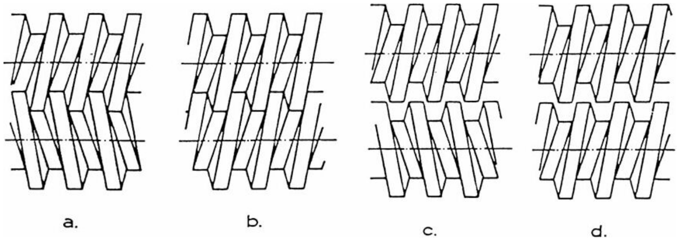


Figure 2: Different kinds of twin screw extruders: a) counter-rotating, intermeshing; b) co-rotating, intermeshing; c) counter-rotating, non-intermeshing; d) co-rotating, non-intermeshing.

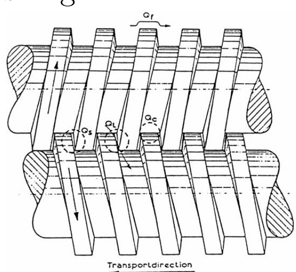


Figure 3: Various leakage flows in the extruder.

Get better axial mixing with a twin-screw than with a single-screw extruder. Important for 2-phase blends.

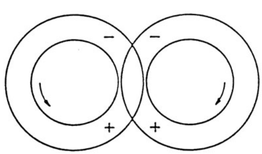


Fig. III.10

Pressure build up in a counter-rotating extruder.

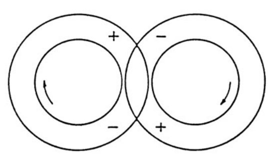


Fig. III.11

Pressure build up in a co-rotating extruder.