Rheometry

FLOW IN CAPILLARIES, SLITS AND DIES DRIVEN BY PRESSURE (POISEUILLE DEVICES)

CAPILLARY RHEOMETER

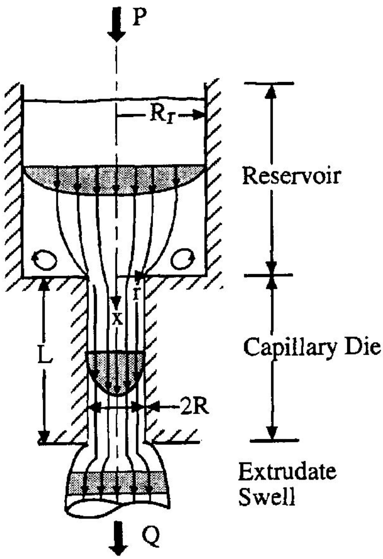

Figure 1: The Capillary Rheometer.

Advantages:
(1) Can operate at high shear rates
(2) May be closer to real processing situation than a rotational rheometer
Disadvantages:
(1) Shear rate is not uniform
(2) Wall slip
(3) Melt fracture
(4) Difficult to clean

Rheometry

CAPILLARY RHEOMETER NEWTONIAN CASE

The velocity profile from the Navier-Stokes Equations is:

$$v(r) = \frac{2Q}{\pi R^2}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{8-9}$$

$$\text{Shear Rate} \qquad \dot{\gamma} = \frac{dv}{dr} \tag{8-6}$$

$$\dot{\gamma}(r) = \frac{-4Qr}{\pi R^4}$$

$$\dot{\gamma}(r) = \frac{r}{R}\dot{\gamma}(R) \tag{8-8}$$

$$\text{Wall Shear Rate} \qquad \dot{\gamma}_w \equiv -\dot{\gamma}(R) \tag{8-11}$$

$$\dot{\gamma}_w = \frac{4Q}{\pi R^3} \tag{8-12}$$

The shear stress from the Navier-Stokes Equations is:

$$\sigma(r) = -\frac{r}{2}\frac{dP}{dz} \tag{8-2}$$

$$\sigma(R) = -\frac{R}{2}\frac{dP}{dz} \tag{8-3}$$

$$\sigma(r) = \frac{r}{R}\sigma(R) \tag{8-4}$$

$$\text{Wall Shear Stress} \qquad \sigma_w \equiv \sigma(R) = \frac{R}{2}\left(-\frac{dP}{dz}\right) \tag{8-5}$$

$$\text{Viscosity} \qquad \eta = \frac{\sigma}{\dot{\gamma}} = \frac{\sigma_w}{\dot{\gamma}_w} = \frac{(-dP/dz)\pi R^4}{8Q} \tag{8-14}$$

Rheometry

CAPILLARY RHEOMETER NON-NEWTONIAN CASE

$$\text{Apparent Viscosity} \qquad \eta_A = \frac{\sigma_w}{\dot{\gamma}_A} = \frac{(-dP/dz)\pi R^4}{8Q} \tag{8-15}$$

Since the shear rate varies across the radius of the capillary, a non-Newtonian fluid will have an effective viscosity that depends on radial position.

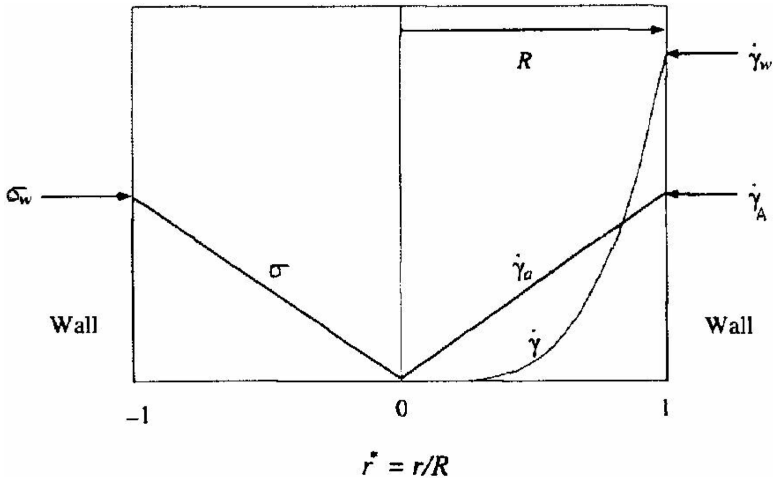

Figure 2: Dependence of Real Shear Stress σ, Apparent Shear Rate $\dot{\gamma}_a$, and Real Shear Rate $\dot{\gamma}$ on Radial Position for a Non-Newtonian Fluid Flowing in a Capillary.

Rheometry
CAPILLARY RHEOMETER
NON-NEWTONIAN CASE
THE RABINOWITCH CORRECTION

There is a unique relation between the wall shear stress and the **apparent** wall shear rate.

$$\text{Wall Shear Stress} \qquad \sigma_w = \frac{R}{2}\left(-\frac{dP}{dz}\right) \tag{8-5}$$

$$\text{Apparent Wall Shear Rate} \qquad \dot{\gamma}_A = \frac{4Q}{\pi R^3} \tag{8-12}$$

This means a plot of $4Q/(\pi R^3)$ against $(-\Delta P)R/(2L)$ makes a universal curve for a given fluid, valid for all flow rates, tube lengths and tube radii.

The Rabinowitch Correction calculates the real wall shear rate from the apparent wall shear rate using such a plot.

$$\dot{\gamma}_w = \left(\frac{3+b}{4}\right)\dot{\gamma}_A \tag{8-20a}$$

$$b \equiv \frac{d(\log \dot{\gamma}_A)}{d(\log \sigma_w)} \tag{8-20b}$$

For a derivation of this, see C. D. Han, Rheology in Polymer Processing, Academic Press (1976) p. 105-106.

In practice, we usually only use a single capillary, so R and L are constants. Thus b can be determined from a plot of Q against $-\Delta P$.

Procedure:

(1) Measure volumetric flow rates Q for many different pressure drops ΔP (or vice-versa).

(2) Determine σ_w and $\dot{\gamma}_A$ from equations (8-5) and (8-12) for each (Q, ΔP).

(3) Plot $\log \dot{\gamma}_A$ against $\log \sigma_w$ and determine $b(\dot{\gamma}_A)$ from the slope via equation (8-20b).

(4) Determine $\dot{\gamma}_w$ from equation (8-20a).

(5) Determine viscosity $\eta = \sigma_w/\dot{\gamma}_w$.

Rheometry

CAPILLARY RHEOMETER

ENTRANCE AND EXIT EFFECTS
THE BAGLEY END CORRECTION

$$-\Delta P = \Delta P_{ent} + \Delta P_{cap} + \Delta P_{ex} = \Delta P_{ends} + \Delta P_{cap} \qquad (8\text{-}47)$$

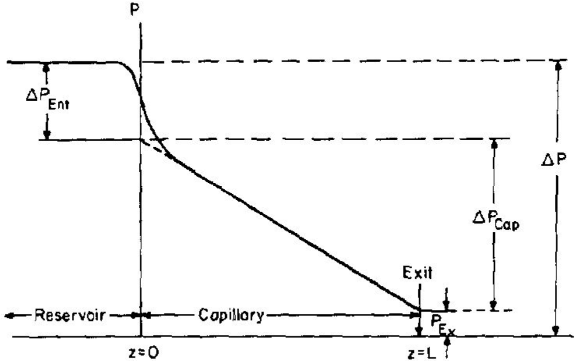

Figure 3: Pressure Distribution in Both the Reservoir and the Capillary.

Considerable energy is dissipated in the entrance region.

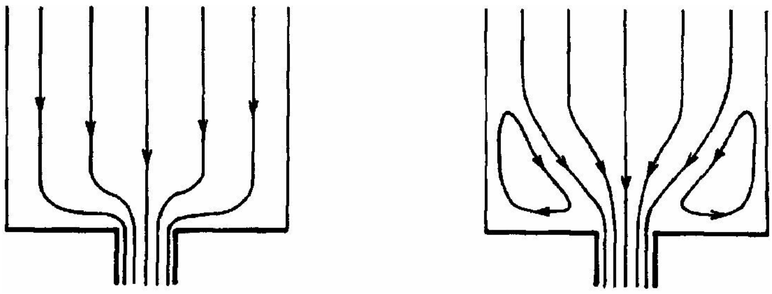

Figure 4: Streamlines in the Entrance Region of a Capillary.

Rheometry

CAPILLARY RHEOMETER

ENTRANCE AND EXIT EFFECTS

THE BAGLEY END CORRECTION

We could measure the pressure distribution in the capillary and directly determine ΔP_{cap}, from which we determine viscosity.

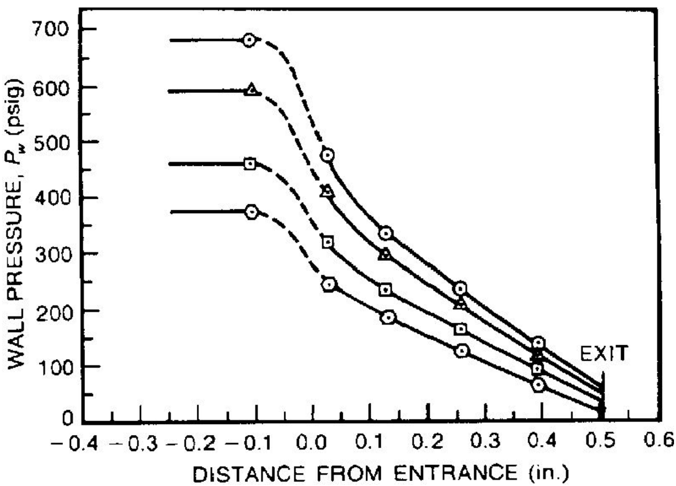

Figure 5: Pressure Distribution in a Reservoir and Capillary for HDPE at Shear Rates of $790\mathrm{s}^{-1}$, $616\mathrm{s}^{-1}$, $313\mathrm{s}^{-1}$ and $160\mathrm{s}^{-1}$.

In practice we simply measure $\Delta P = P_d$ the driving pressure and the volumetric flow rate Q.

Rheometry
CAPILLARY RHEOMETER
ENTRANCE AND EXIT EFFECTS
THE BAGLEY END CORRECTION

If we use data from a number of different length capillaries, we can separate ΔP into ΔP_{ends} and ΔP_{cap}.

$$\Delta P_{cap} = \frac{2\sigma_w L}{R}$$

$$\Delta P = P_d = \Delta P_{cap} + \Delta P_{ends} = \frac{2\sigma_w L}{R} + \Delta P_{ends}$$

$$\text{Define End Correction} \qquad e \equiv \frac{\Delta P_{ends}}{2\sigma_w} \tag{8-49}$$

$$P_d = \frac{2\sigma_w L}{R} + 2\sigma_w e$$

$$\sigma_w = \frac{P_d}{2(L/R + e)} \tag{8-48}$$

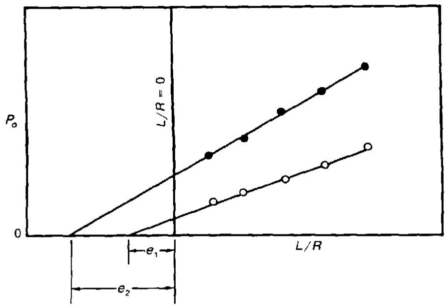

Figure 6: Bagley End Correction for Capillary Flow.

Bagley treats the extra pressure drops at the ends as though the capillary were longer than it really is by $L/R \to L/R + e$.

CAPILLARY RHEOMETER

ENTRANCE AND EXIT EFFECTS

THE COGSWELL ORIFICE SHORT-CUT

The Bagley procedure is still to much work to be used routinely, because experiments must be done with at least three different dies to obtain accurate data. Cogswell has suggested the following shortcut:

(1) Use one long die ($L/R > 30$)

(2) Use an orifice ($L/R \cong 0$)

The driving pressure for the orifice P_d^0 is taken to be ΔP_{ends}.

$$\sigma_w = \frac{(P_d^L - P_d^0)R}{2L} \tag{8-52}$$

SUMMARY

1. Measure P_d and Q for different capillaries (L/R).
2. Determine Apparent Wall Shear Rate $\dot{\gamma}_A$ from Q.

$$\dot{\gamma}_A = \frac{4Q}{\pi R^3}$$

3. Determine Wall Shear Stress from P_d at each $\dot{\gamma}_A$

either from (8-52) or from Bagley plot: $\sigma_w = \dfrac{P_d}{2(L/R + e)}$ (8-48)

4. Determine Real Wall Shear Rate $\dot{\gamma}_w$ from

Rabinowitch correction: $\dot{\gamma}_w = \left(\dfrac{3+b}{4}\right)\dot{\gamma}_A$ with $b \equiv \dfrac{d(\log \dot{\gamma}_A)}{d(\log \sigma_w)}$ (8-20)

Rheometry
CAPILLARY RHEOMETER

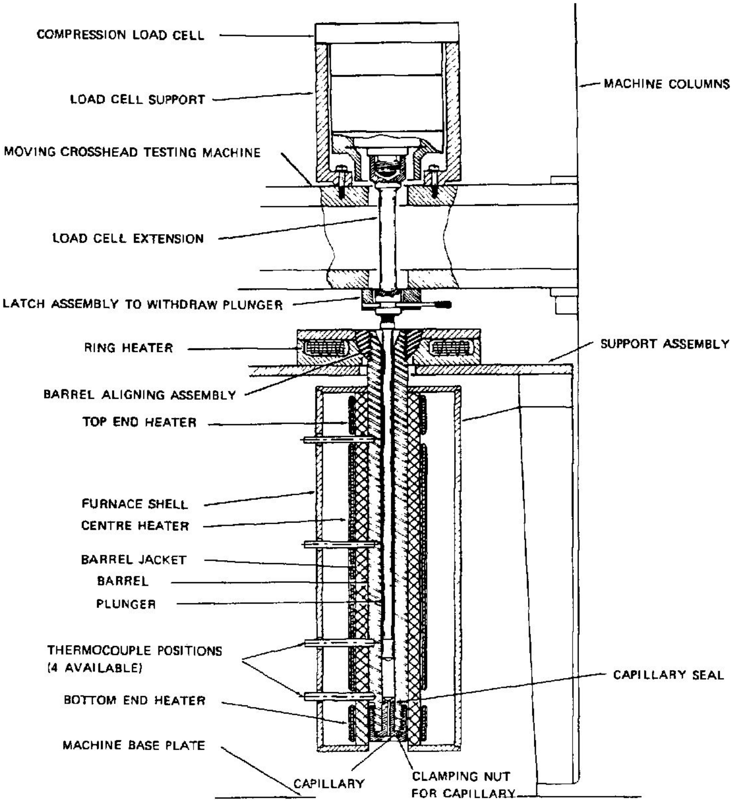

Figure 7: Instron Capillary Rheometer.

$$P_d = \frac{F_d}{\pi R_b^2} \tag{8-57}$$

$$Q = \pi R_b^2 V_b \tag{8-24}$$

V_b is the plunger velocity, R_b is the barrel radius,
F_d is the force measured in the load cell.

GRAVITY-DRIVEN CAPILLARY RHEOMETERS

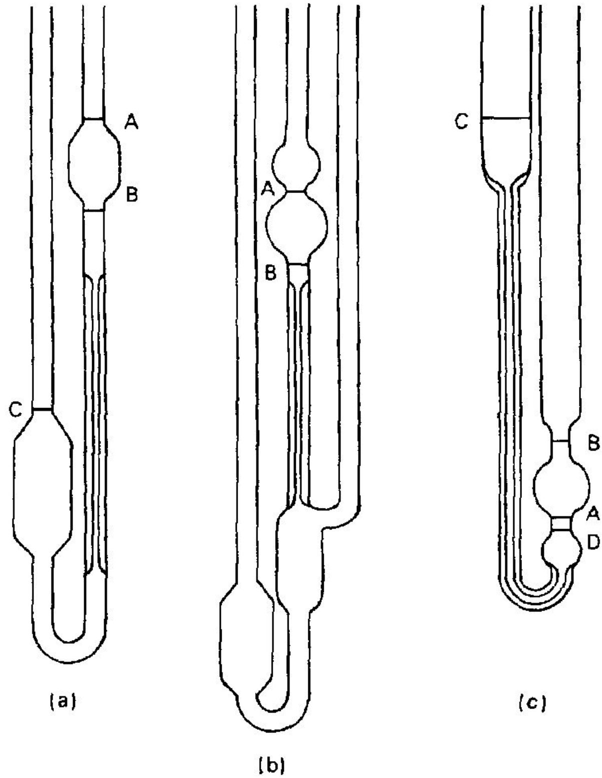

Figure 8: Glass Capillary Viscometers for Dilute Polymer Solutions (and other low viscosity Newtonian liquids). (a) Ostwald design, (b) Cannon-Ubbelohde design, (c) Reverse Flow design.

These viscometers use capillaries, but the pressure decreases over the course of the experiment.

The flow time (for meniscus to travel from A to B) is proportional to the viscosity. The viscometers are calibrated with standards and measure viscosity to four significant figures.